



## THERMO-MECHANICAL STRESS ANALYSIS FUNCTIONALLY GRADED TAPERED SHAFT SYSTEM BY USING ANSYS

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### ABSTRACT

*Present work deals with the study of stresses developed in tapered functionally graded (FG) shaft system under both thermal and mechanical environment for three noded beam element by using Timoshenko beam theory. The temperature distribution in radial direction is assumed based on one dimensional steady state temperature field by Fourier heat conduction equation without considering heat generation. Temperature dependent material properties are varied along the radial direction using power law gradation. Tapered FG shaft consists of rigid disk attached at its centre and shaft is mounted on two flexible bearings acts as spring and damper, inner radius of the tapered shaft is varying in x direction keeping thickness of hollow tapered shaft is constant. For the present analysis the Mixture of Stainless steel (SUS304) and Aluminium oxide (Al<sub>2</sub>O<sub>3</sub>) are considered as inner and outer surface material of the FG shaft. Three dimensional constitutive relations are derived based on first order shear deformation theory (FSDT) for Timoshenko beam element considering rotary inertia, strain and kinetic energy of shaft and gyroscopic effect. In present study, structural and hysteretic damping is incorporated. Hamilton's principle is used to derive governing equation of motion for three noded beam element for six degree of freedom per node. Complete MATLAB code is generated and shows that temperature field and power law gradient index have important part on material properties. Comparative study is carried out for Stainless steel and FG tapered shaft, shows that stress developed in FG shaft is comparatively lower than Steel shaft. Various results are obtained for coupled and uncoupled environment. Transient stress is obtained for*

*varying power law index value and speed as a parameter. Stress amplitude increases for increase in speed and power law index. Results achieved for FG shaft shows advantages over steel shaft. The work extends to optimise for better results by using advanced simulation software ANSYS 14.0 workbench. Comparison results are tabulated and optimised results are interpreted for future extension.*

**Keywords:** Functionally graded materials (FGMs), Power law index, Tapered shaft, Timoshenko beam theory (TBT), Three noded beam element, Finite element method, Thermo-mechanical, Stress analysis, ANSYS 14.0.

### INTRODUCTION

Composite materials are materials, composed of two or more fundamental materials with different properties, when combined to get a material with different properties than that of individual constituents. Composite material structures are more frequently used in engineering fields as their high strength to weight ratio and high stiffness to weight ratio is basically favourable for material selection. Main disadvantage with composite material is, weakness in interface between neighbouring layers, which is popularly known as delamination phenomenon that may cause structural failure. To overcome this problem, a new class of material presented, named as Functionally Graded Materials (FGMs). FGMs are recognised as, whose material



properties are varying in certain direction and thus overcome interface weakness. FGMs are defined as, the materials whose volume fractions of two or more materials are varied continuously along certain direction to attain required purpose. FGMs provide better material response and excellent performance in thermal environments like thermal barrier and space application, where it is used to protect space shuttle from heat generated during re-entry to Earth's atmosphere by modelling ceramic material at outer surface metal at inside surface.

Rotor dynamics has a significant history, mainly due to its relationship with theory and practice. Rotor dynamics is a particular branch of applied mechanics deals with the performance and analysis of rotor assemblies. Rotor dynamics mostly used to analyse the performance of a turbine shafts, jet engine to auto engines and computer storage disks. Basically Rotor dynamics deals with rotor and stator. Rotating part in mechanical devices are called rotors, which are supported on bearings, thus shaft rotate freely about its axis. Many engineering components are deals with the subject of Rotor dynamics and which gives better solution for components like turbines, compressors, alternators, blowers, motors, pumps, brakes etc.

Rotor delivers with behaviour of materials to limit their spin axis in a more or rigid way to a fixed position in space, those are mentioned to as fixed rotor (considering spin speed is constant), while rotors which are not considering in any way are mentioned as free rotors (considering spin speed is governed by conservation of angular momentum). In process, Rotors have excessive deal with

rotational energy and small amount of vibrational energy.

Aurel B. Stodola (1924) developed dynamics of elastic continuous rotor having discs without considering gyroscopic moment, balancing of shaft, secondary resonance phenomenon due to gravity effect and methods to determine critical speeds of shafts for variable cross sections, also by using Coriolis accelerations supercritical solutions can be stabilized.

Baker (1933) found and defined that because of contact between rotor and stator system exhibits self-excited vibrations. David M. Smith (1933) found formulas for predicting threshold spin speed for supercritical instability varied through bearing stiffness and also with ratio between external to internal viscous damping. Many variations came closer to practical needs of the rotor dynamic field for Jeffcott rotor model. Prohl's and Myklestad's (1945) analysed instabilities and modelling methods in rotors dynamics by Transfer Matrix Method (TMM).

In 1960s, for exact solution capabilities, numerical methods are established for structural dynamic analysis, rotor dynamics codes and digital computer codes were constructed on TMM method. In 1970s alternative fundamental procedure developed that is Finite Element Method (FEM), developed for solution of beam type of models. In 21<sup>st</sup> century, rotor dynamics are combined FEM and solids modelling methods to create simulations that adapt the coupled behaviour of disks, elastic shafts and elastic support assemblies into a single, multidimensional-model.

## **OBJECTIVES**



1. MATLAB results taken from the literature review of before work done by dinesh
2. Modelling has been carried of in creo2.0 modelling software
3. Literature work results has been interpret in advanced simulation software ANSYS 14.0 for optimum results.

## LITERATURE REVIEW

Great number of research has been done in the field of modelling and analysis of FGM's. Some of important workout has been done and presented in following section. Composite materials possess high strength and stiffness. This is an outcome for the use of composite materials in field of aircraft and space shuttles. In an attempt to develop heat resistant materials FGMs are developed. Composition of two or materials and structures changes over volume by using some gradation laws such as, exponential law power law, stepwise variation and continuous variation [1].

Schmauder et al. [2] investigated mechanical behaviour of ZrO<sub>2</sub>/NiCr 80 20 compositions FGMs are analysed and compared with experimental results. And also found that new parameter matrixity controls the stress level of composite, globally and also locally. Sladek et al. [3] analysed time dependent heat conduction in nonhomogeneous FGMs. Laplace transforms technique is used to solve initial boundary value problem. Results obtained for finite strip and hollow cylinder having exponential variation of material properties. Shao et al. [4] presented stress analysis of FG hollow circular cylinder in combined mechanical and thermal environment by considering linearly increasing temperature. Temperature dependent material properties are considered

and solution for ordinary differential equations are solved by Laplace transforms technique. Farhatnia et al. [5] presented stress distribution for composite beam having FGM in middle layer. Temperature dependent material properties are considered for uniform temperature gradient. Jyothula et al. [6] presented nonlinear analysis of FGMs in thermal environment by changing material variation parameter, aspect ratio, and boundary condition re analysed with higher order displacement model. Nonlinear simultaneous equation are obtained by Navier's method and equations are solved by Newton Raphson iterative method. Callioglu [7] presented thermoelasticity solution for FG disc. By using infinitesimal deformation theory and power law distribution used to get solution. Stress and displacement variation are presented along radial position due to centrifugal action, steady state temperature, internal and external pressure. Abotula et al. [8] studied stress field for curving cracks in FGMs for thermo-mechanical loading.

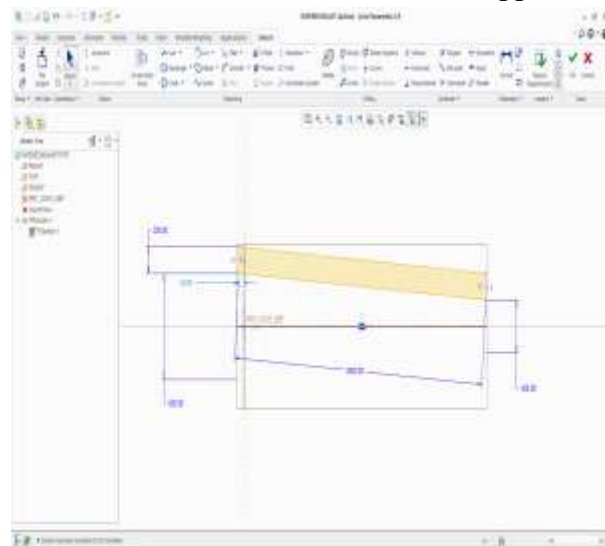
Using strain energy density criterion effect of curvature parameters, temperature gradients on crack growth directions, non-homogeneity values are found and discussed. Bhandari et al. [9] studied parametric study of FGM plate by varying volume fraction distribution and boundary conditions. Static analysis of FGM plate has studied by sigmoid law and compared with literature. Kursun et al. [10] presented stress distribution in a long hollow FG cylinder under thermomechanical environment. By using infinitesimal deformation theory, solution for displacement model are found.

## MATERIAL MODELLING FOR TAPERED FG SHAFT

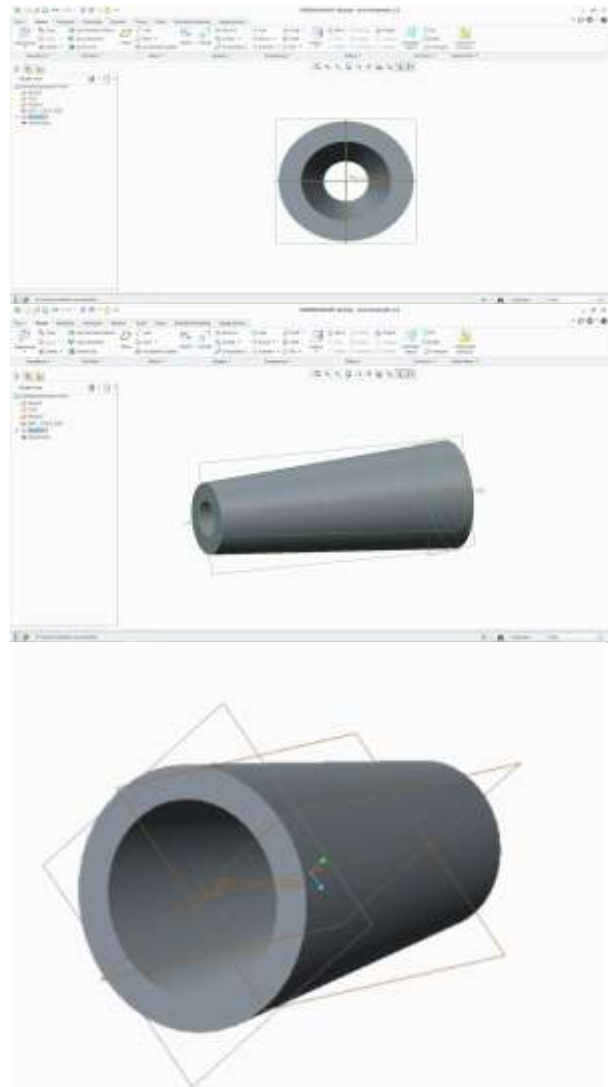
Material modelling of FG tapered shaft is explained in detail in this chapter by taking power law gradation and exponential gradation.

### Actual Material Properties of FGM

Properties of FGMs are changing along certain directions, so that it is required to find effective material properties of FGMs applying to shaft. For exact analysis of FGMs it is required to find accurate material properties. Many simulations are established for determining properties of FG shaft. Bulk constituent properties assumes no interaction between phases by employing rule of mixture. Thermo physical properties are derived by variation approach. Spatial distribution of constituent materials are having information about micromechanical approach.



### 3D MODELLING



## RESULTS AND DISCUSSION

A complete MATLAB code has been developed and validated for above formulation. Results are presented for different stresses, speed and power law index value based on problem specified below.

### Problem Description and Summarization of Discussion

Tapered shaft consists of rigid disk attached at its centre, supported on two similar bearings. Shaft is modelled for present analysis using three noded beam element. Dimensions of shaft and rigid disk are presented in Table 5. 1. Tapered shaft is divided into 10 elements, and 8 equal thick layers. Stiffness and



damping coefficients for two identical bearings are taken as  $K_{yy}=7 \times 10^7$  N/m,  $K_{zz}=5 \times 10^7$  N/m,  $C_{yy}=700$  Ns/m,  $C_{zz}=500$  Ns/m.

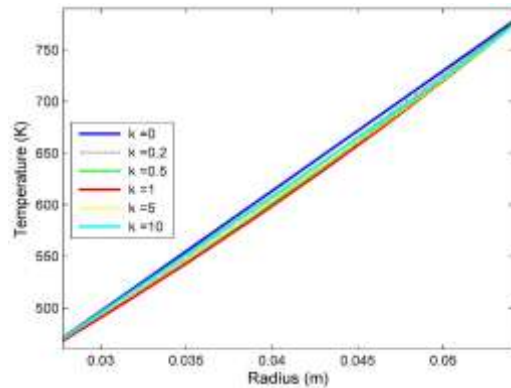
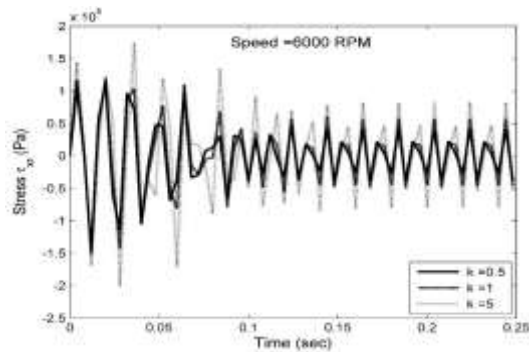
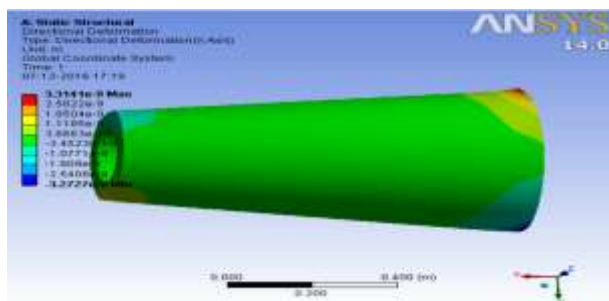


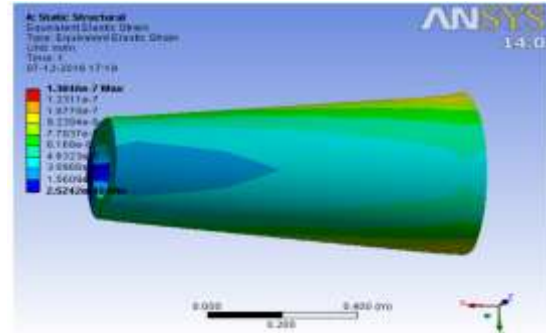
Figure: Temperature variation in mid-section of tapered FG shaft.



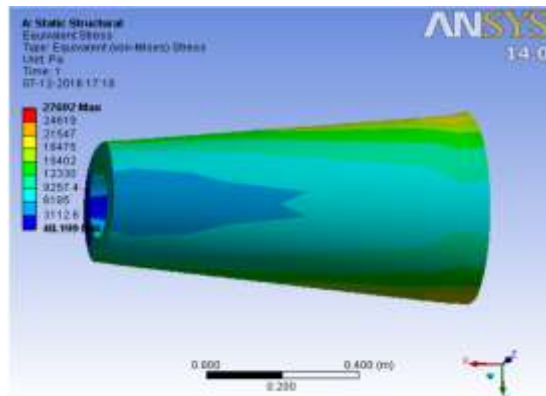
## ANSYS RESULTS



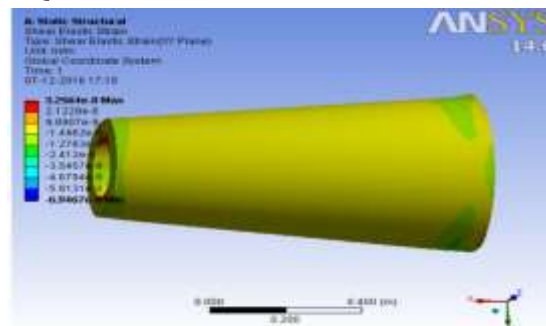
DIRECTIONAL DEFORMATION



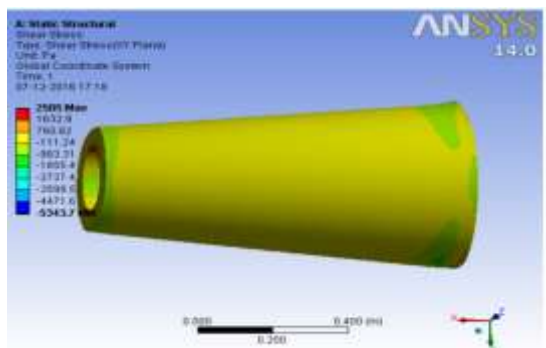
ELASTIC STRAIN



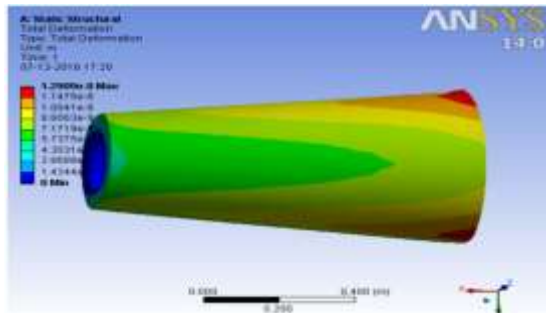
EQUIVALENT STRESS



SHEAR ELASTIC STRAIN



SHEAR STRESS



TOTAL DEFORMATION

### Conclusions

Present study supports to draw following important conclusions.

- i. Three noded Timoshenko beam element has been implemented for modelling and analysis of FG tapered shaft by taking into account of structural damping and hysteretic damping in temperature environment.
- ii. The temperature distribution is assumed based on one Dimensional steady state temperature field by using Fourier heat conduction equation without considering heat generation.
- iii. Temperature dependent material properties are established by taking different power law index value.
- iv. Stress values are compared between steel and FG shaft by taking temperature dependent material properties for linear variation of temperature, it is found that stresses developed in FG shaft is lower than Steel shaft. Also for better results we use the ANSYS software for comparing results.
- v. It is also noted that stress increases as power law index increases.
- vi. Stress amplitude increases as speed of the shaft increases.

### Scope of future work

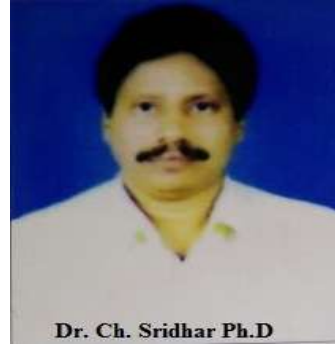
- i. Nonlinear modelling of FG shaft.
- ii. Active vibration control of FG shaft.
- iii. Analysis and control of breathing crack in FG shaft.

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