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FREE VIBRATION AND THERMAL ANALYSIS OF RANDOMLY ORIENTED CARBON NANO TUBE BASED FUNCTIONALLY GRADED BEAM

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ABSTRACT

Modern technology demands materials having improved mechanical, thermal and chemical properties which must sustain the different environmental conditions. The carbon nanotube (CNT) reinforced functionally graded materials (FGM) are expected to be the new generation materials having wide range of unexplored potential applications in various technological areas such as aerospace, defence, energy, automobile, medical, structural and chemical industry. Present work deals with the finite element modelling and free vibration analysis of CNT based functionally graded beam using three dimensional Timoshenko beam theory. It has been assumed that the material properties of CNT based FG beam varies only along the thickness and these properties are evaluated by rule of mixture. The extended Hamilton's principle has been applied to find out the governing equations of CNT based FG beam. Finite element method is used to solve governing equation with the exact shape functions. Natural frequencies are calculated and validated with available literature. The convergence study has been carried out with obtained results. The effect of variation of CNT volume fraction and boundary conditions on the natural frequency of the beam has also been studied. Initial analysis deals with CNTs assumed to be oriented along the length direction only. But practically it is not possible. So, further work deals with the free vibration analysis of functionally graded nano composite beams reinforced by randomly oriented straight single walled carbon nanotubes (SWCNTs). The Eshelby–Mori–Tanaka approach based on an equivalent fiber is used to investigate the material properties of the beam. The equations of motion are derived by using Hamilton's principle. Results are presented in tabular and graphical forms to show the effects of carbon nanotube orientations, slenderness ratios and boundary conditions on the dynamic behaviour of the beam. Mainly, Functionally Graded Materials (FGMs) are invented for high temperature applications such as Aerospace and Nuclear industries. Moreover, Carbon Nano Tubes (CNTs) are reinforced with FGM to improve their thermal properties along with Mechanical and Electrical properties. So, next important work is the thermal analysis of Carbon Nano Tubes based FG (FG-CNT) Timoshenko beam. Different types of temperature distributions are used to find out the thermal behaviour of Timoshenko beam. However, the material property distribution is assumed to be exponential along the thickness direction for exponential type temperature distribution, linear and power law type material distribution for linear type temperature distribution. First order Shear Deformation Theory (FSDT) is implemented along with plain strain condition to formulate expressions for stresses and strains. Finally, effect of linear and exponential type of temperature distributions on Young's modulus, total stresses and strains is evaluated numerically for given beam. Finite difference method is also applied to find out temperature distribution for power law material distribution. Same thermal analysis is also carried out in this case also. The effect of CNT orientation on stresses and strains has also been found out for exponential distribution of material and temperature. The comparison of these cases will lead to the conclusion and it will serve the purpose of presented work.

INTRODUCTION AND LAYOUT OF PRESENT WORK

Functionally Graded Material (FGM) belongs to a class of advanced material characterized by variation in properties as the dimension varies. FGM was invented with the prime requirement of thermal stability at high temperatures along with better mechanical properties. FGM concept was originated in Japan in 1984 during the space plane project. Metallic part of FGM will take care of better mechanical properties and ceramic part will take care of thermal stability at high temperatures. The overall properties of FGM are unique and different from any of the individual material that forms it. Due to these advantages FGM has a wide range of applications such as in

 Aerospace to make space plane bodies, components of the rocket engines Medicine to replace the living tissues like bones and teeth Defence as a penetration resistant material for bullet proof vests as well as armour plates Energy as thermal barrier, protective blade in gas turbine Optoelectronics as graded refractive index material Other applications consist of automobile engine component and nuclear reactor components etc.

These applications explicitly require good thermal properties. So, it is important that analysis of thermal behaviour of FGM should be carried out which has initiated the work related to this area. Lot of researchers have presented their work so far.

Important advantages of CNT based FGM:

1. Higher strength to density ratio 2.Higher stiffness to density ratio 3. Better fatigue and wear resistance 4. Better elevated temperature properties (Higher strength-Lower creep rate) 5.Ability to fabricate directional mechanical properties 6. Provide multi functionality 7. Provide ability to control the deformation, dynamic response of the system, wear and corrosion of parts etc. 8. Provide ability to remove stress concentrations 9. Provide opportunities to take the benefits of different material systems

The key of using CNT based FGM is that one can obtain these properties as per the requirement just by varying the distribution and composition of CNT. That's how one can get directional properties and can control other parameters. Another advantage stated above is the stress concentration free material. It is because, the cross-section shows there are no layers inside the material and instead there is a continuous gradation of materials from top to bottom. So, there is no stress concentration and delamination of layers. As it can be manufactured from CNT, polymers, ceramics, one can take the advantages of each constituent material.

Layout of present work: Present work is divided into three chapters as Free Vibration Analysis, Effect of CNT Orientation and Thermal Analysis. First chapter deals with finding out the material properties of the beam which is assumed to vary along the thickness. Later finding out the translational and kinetic energy of the Timoshenko beam, applying Hamilton's theorem to find out governing equation and solving it by using Finite element analysis to obtain equation of motion. Present theory is validated first and then effect of CNT volume fraction and slenderness ratio is obtained. Dynamic analysis is also carried to find out the displacement response. Second chapter deals with the effect of CNT orientation. Due to CNT orientation Young's modulus of the material changes which further affects the natural frequency of the system. This effect is obtained for different CNT volume fractions and slenderness ratios. Third chapter deals with thermal analysis of CNT based FG Timoshenko beam. Different types of thermal laws are applied with different material property variation to study the combined effect of thermal and material property variation on thermal stresses and strains. CNT orientation effect on the thermal stresses and strains is also obtained. Later finite difference method is applied to find out temperature distribution along the beam. That is also tested for finding out the effect of CNT orientation. The comparison between each case is presented graphically. Finally, based on the output from each chapter conclusions are drawn which will be helpful for using CNT based FGM in practical situations.

Motivation and objective of the work : Materials are invented based on the need for the application. Initially, metals and non-metals were invented, later polymers and ceramics. In this sequence, with the prime requirement of light weight and high strength, composites are invented. To improve the thermal stability at high temperature, FGMs are invented. During this evolution of materials, CNTs are introduced with excellent mechanical, thermal and electrical properties. Then a new concept is evolved to take advantages of both CNT and FGM is CNT based FGM. In current scenario, there is a lot of scope for research in this area. The study of material properties, mechanical properties, different types of analyses can be carried out on CNT based FGM to put it into the practical applications. Presently there are some areas where this material is being used, but by looking at the advantages of using this material, it should be used widely. With this motivation, present work is carried out. The objectives of the present work are as stated below:

1. Finding out the material properties of CNT based FGM

2. Free Vibration Analysis of Timoshenko beam

3. Finding out the effect of CNT orientation

4. Thermal Analysis by analytical and finite difference method

Free vibration analysis: Free vibration is the basic type of vibration in which the system vibrates with its natural frequency. The free vibration analysis carried out on CNT based FG.

MODELLING OF BEAM

Calculation of Mechanical Properties of Beam: The modelling of beam starts with the calculation of material properties. As CNT volume is assumed to vary along the thickness only, material properties for each layer are calculated first and finally effective values for the entire beam are calculated. Here, linear variation of volume fraction of CNT (V_{cnt}) is considered. It is calculated by following formula,

$$V_{cnt} = \left(1 - \frac{2z}{h}\right) V^{*}_{cnt} \qquad (1)$$

Here, V^{*}_{cnt} depends on mass fraction and density of CNT and density of matrix and it is given by

$$V^{*}_{cnt} = \frac{\Lambda_{cnt}}{\Lambda_{cnt}} + (\rho^{cnt} / \rho^{m}) - (\rho^{cnt} / \rho^{m})\Lambda_{cnt}$$

where Λ_{cnt} called as is mass fraction of CNT. For uniform CNT distribution V_{cnt} equals to the *cnt V* . After calculating volume fraction Young' modulus, Shear modulus, Poisson's ratio and Density of a beam can be calculated as follows,

$$\left.\begin{array}{ll} E=\eta_1 V_{cnt}E^{cnt}+V_m E^m & \dfrac{\eta_2}{G}=\dfrac{V_{cnt}}{G^{cnt}}+\dfrac{V_m}{G^m} \\ \upsilon=V_{cnt}\upsilon^{cnt}+V_m\upsilon_m & \rho=V_{cnt}\rho^{cnt}+V_m\rho_m \end{array}\right\} \quad (2)$$

where, *cnt E* & *cnt G* are Young's modulus in longitudinal, transverse direction & shear modulus for carbon nano tube. E*m E ,m G* are Young's modulus & shear modulus for matrix(ceramic).*j* (j = 1, 2 and 3) is CNT efficiency parameter.

FINITE ELEMENT ANALYSIS

To solve the above governing equation Finite element analysis is implemented. Finite element analysis aim is to find out the field variables (displacement) at nodal points by approximate analysis. These field variables are related to the field variables of nodal points inside the element. Here employing the approximate solution method the governing equations are approximated by a system of ordinary differential equations. The solution of the above partial differential equations is assumed in the following form

$$u_0=c_1+c_2x+c_3x^2,$$
$$w_0=c_4+c_5x+c_6x^2+c_7x^3,$$
$$\phi=c_8+c_9x+c_{10}x^2$$

In above equation, the order of interpolation of slope is less than that of order of transverse displacement. This is to avoid the shear locking of the element. Some constants from above equation can be defined as

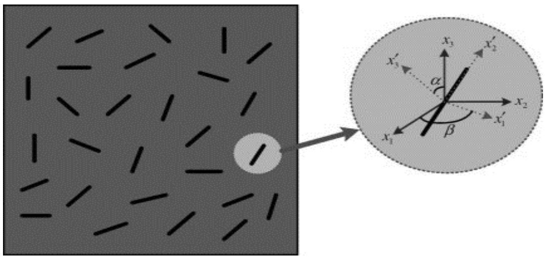
$$c_3=\frac{\mu(c_8-c_5)}{2},\quad c_7=\frac{\eta(c_8-c_5)}{6},\quad c_6=\frac{c_9}{2},\quad c_{10}=\frac{\eta(c_8-c_5)}{2}$$

EFFECT OF CARBON NANO TUBE ORIENTATION:

Material properties of fg-cntrc:

Mechanical property	Equivalent fibre[9]
Longitudinal Young's modulus (E_{LEF})	649.12 (GPa)
Transverse Young's modulus (E_{TEF})	11.27 (GPa)
Longitudinal shear modulus (G_{EF})	5.13 (GPa)
Poisson's ratio (υ_{EF})	0.284

Modelling of beam: The effect of randomly oriented, straight CNTs is formulated here. The orientation of a straight CNT is characterized by two Euler angles α and β, as shown in Figure



Representative volume element (RVE) with randomly oriented, straight CNTs

RESULTS AND DISCUSSION

This theory has been implemented to a beam with h = 1 m, L/h = 20 to study the convergence of non-dimensional fundamental frequency for V^*_{cnt} = 0.075. It is calculated by following formula:

$$\lambda^2 = \omega L^2 \sqrt{\frac{\rho_m A}{E_m I}}$$

Now, with the material properties assumed for the beam are as mentioned in the *table*

Material Property	CNT	Matrix
Young' Modulus (GPa)	900	10
Poisson's Ratio	0.28	0.3
Density (Kg/m^3)	2100	1150

With these properties and applying the present theory, results of the first five non dimensional frequencies for clamped–clamped (C–C) are obtained. The effect on SFG-CNTR beam based on Timoshenko beam theory with different number of elements is obtained to study the convergence and is shown in *table*. Graph is also plotted to observe the convergence as shown in *figure*. It is observed from above *figure* and *table* that the convergence of the present results occur with a number of element N = 100.

Mode No.	20	40	60	80	100
1	5.7046	5.4562	5.4229	5.4089	5.4030
2	7.7632	7.7280	7.6686	7.6506	7.6413
3	9.6638	9.4572	9.3968	9.3708	9.3599
4	9.8956	10.9441	10.8519	10.8237	10.8090
5	11.7088	12.2242	12.1418	12.1037	12.0874

Effect of number of elements on non-dimensional fundamental frequency

Thermal analysis: Thermal analysis for the CNT based FG Timoshenko beam is carried out by two methods. First is by considering analytical method for finding out the temperature distribution and second is FDM for finding out the temperature distribution. The modelling of the beam, effect of temperature distribution on strains and stresses by both these methods and effect of CNT orientation.

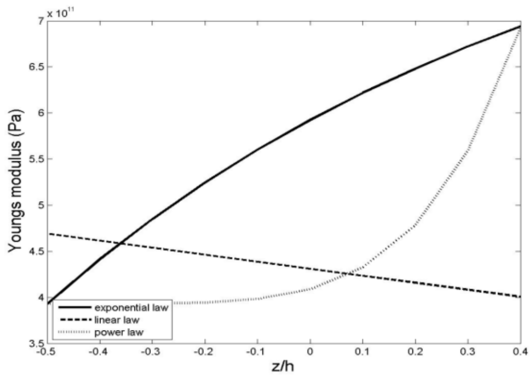
z/h	Non-dimensional Stresses	
	Reference	Present Case
-0.5	0.85	1.1
-0.3	0.1	0.2
-0.1	-0.3	-0.4
0.1	-0.35	-0.37
0.3	0	0.02
0.5	1.0	1.0

After validation, results obtained for different cases are presented in the following subsections.

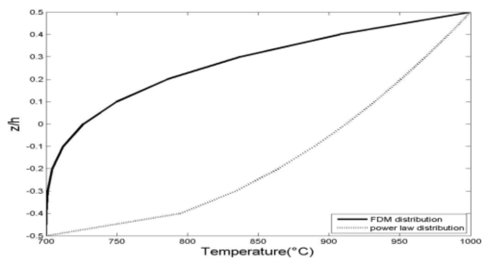
Numerical Analysis: The theory presented above is applied to the beam with the simply supported end conditions and rectangular cross section with width (b) = 0.4 m, thickness (h) = 1 m. The required material properties are given in Table

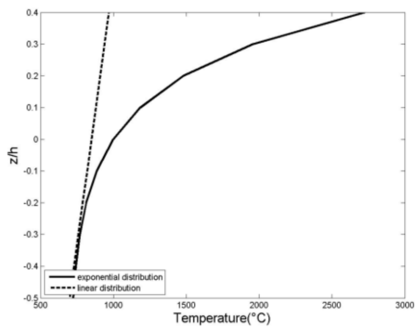
Material	$E(GPa)$	$\rho(Kg/m^3)$	ν	$k(W/m^{\circ}K)$	$\alpha(/^{\circ}K\times10^{-6})$
SWCNT	900	2100	0.28	3500	5.1
Al_2O_3 [1]	393	3970	0.3	30.1	8.8

The shaft is divided into *10* layers along the thickness. Using above properties graph is plotted to observe the variation of Young's modulus along the thickness direction for exponential, linear and power law variation in material properties. The temperature boundary conditions considered here are,



Variation of Young's modulus along the thickness for exponential, linear and power law material distribution

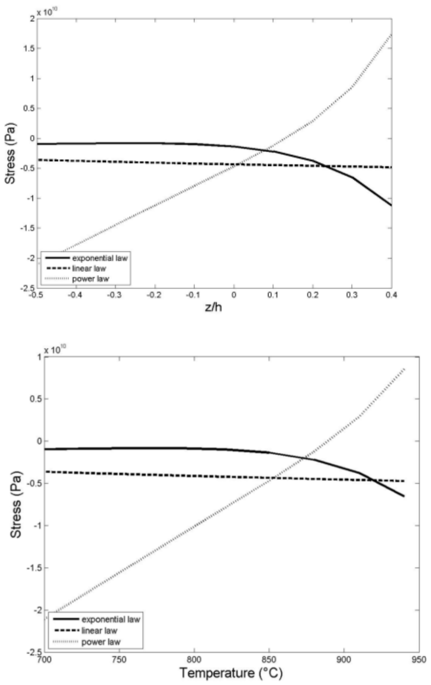




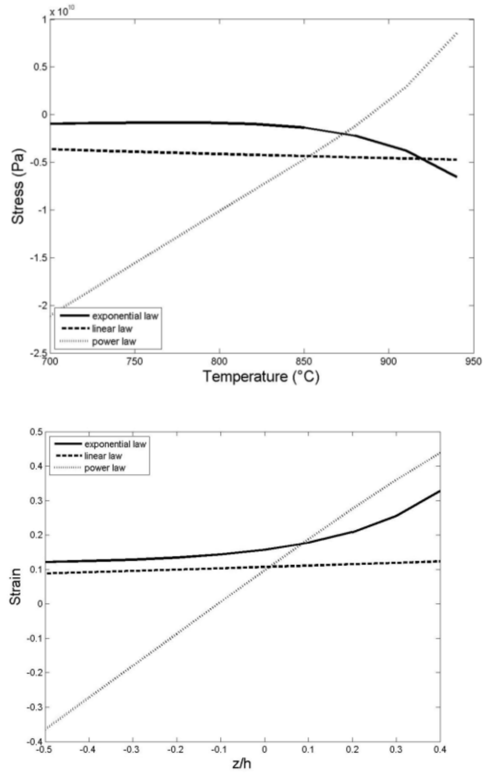
Comparison of temperature distribution along the thickness according to FDM and power law

PRESENT RESULTS

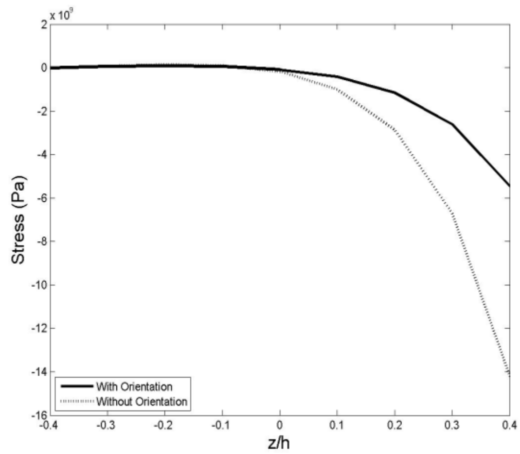
As per the formulation and numerical analysis, the results are obtained for all cases. For first case, exponential temperature distribution is applied to the beam having exponential variation in material properties and plotted graphically. For second case, the linear temperature distribution is applied to the beam having linear variation in material properties. The value of stress is almost constant because of its low thermal conductivity and less co-efficient of thermal expansion. The nature of variation of stresses will be according to the material properties i.e. for exponential variation of material properties the stress varies exponentially and so on. It is observed that for layer by layer analysis, the exponential material property variation, stresses are compressive in upper half and tensile in lower half of the beam; for linear material property variation stresses are compressive throughout; for power law material property variation stresses are tensile in upper half and compressive in lower half of the beam.

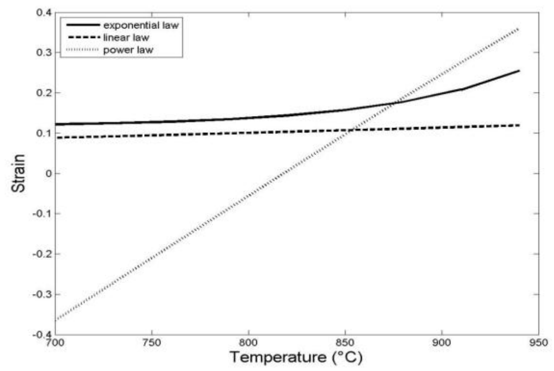


Stress distribution along the thickness for exponential material and temperature distribution, linear material distribution and linear temperature and power law material distribution

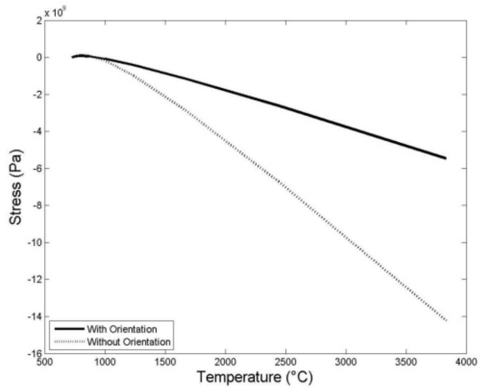


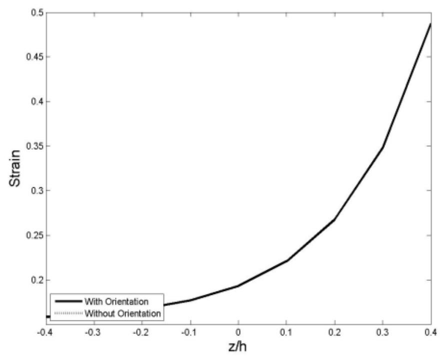
Stress distribution and Strain variation according to the temperature for exponential material and temperature distribution, linear material distribution and linear temperature and power law material distribution.





Effect of CNT orientations on stress distribution along the thickness for exponential material and temperature distribution





Effect of CNT orientations on stress distribution according to the temperature for exponential temperature and material distribution

CONCLUSIONS AND FUTURE SCOPE

In the present work, CNT based FG Timoshenko beam has been modelled using finite element method. Firstly convergence of results has been studied and then different analyses such as static and dynamic analyses have been carried out. Non-dimensional fundamental frequencies are calculated for different volume fractions of CNT and slenderness ratio. Mode shapes and displacement diagrams are also plotted for this beam. From this analysis it can be concluded that

- As volume fraction of CNT increases, fundamental frequency also increases. The reason behind this is more volume of CNT provides more stiffness to the beam which results in higher frequencies.

- Non-dimensional fundamental frequency increases as slenderness ratio increases.

- The response of displacement versus Time shows that for increasing values of volume fraction displacement goes on reducing.

- The maximum displacement is obtained at the centre of the beam for simply supported case.

In the analysis of thermal behaviour thermal stresses and strains are evaluated numerically and plotted graphically for different cases with linear, exponential and power law temperature variation. Temperature distribution is also calculated by using FDM. That is compared with stresses and strains found by considering power law for temperature distribution. Later the effect of CNT orientation has been obtained on stresses and strains with exponential material and temperature distribution. These cases have led to following conclusion.

- The nature of variation of stresses is greatly affected by the material property variation assumed.

- The nature of variation of strains is greatly affected by the temperature distribution assumed.

 Better thermal behaviour including both stresses and strains can be obtained if the temperature distribution and material property variation are similar.

 The temperature distribution is different in both cases i.e. calculated by using FDM and power law.

 As CNT volume fraction increases, considerable changes in thermal behaviour of the beam are observed.

 Response for change in stresses and strains of matrix material is very low because of their low thermal conductivity.

 The stress and strain variation is different for FDM and power law cases but the range of variation for strains and stresses is same.

 Stress variation with temperature distribution calculated by using FDM does not vary much compared to that of calculated by power law but strain varies greatly for these two cases.

 Due to randomly oriented CNT there is reduction in Young's modulus of the beam. This reduction has led to corresponding reduction in stresses. But it is applicable for high volume fraction of CNT.

 There is no effect of CNT orientation on strain distribution either along the thickness or temperature.

So, the work presented in this paper will be useful for selecting the temperature distribution for the particular application. It will enable the design engineer to choose a particular temperature distribution so as to minimise the stresses and strains. Also, it can be concluded that FDM can be used over to other analytical methods as it is easy to implement and gives quite good results.

FUTURE SCOPE

 Vibration analysis under geometric nonlinearity Vibration analysis of a non-uniform CNT based FG beam Active vibration control of Timoshenko beam CNT based FG beam under thermo-mechanical loading Buckling analysis of non-uniform CNT based FG beam under thermal loading

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