QUANTUM MECHANICS IN RELATION WITH GALILIEN TRANSFORMATION

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ABSTRACT

This paper considers a area with Generalized Uncertainty precept (GUP) which can be obtained inside the body of the deformed commutation relations. inside the area with GUP-This found adjustments referring to coordinates and instances of shifting and relaxation frames of reference in the first order over the parameter of deformation. within the non-relativistic case we discover the deformed Galilean transformation which is rotation in Euclidian area–time. this variation is much like the Lorentz one however written for Euclidean area–time where the speed of light is changed via some speed associated with the parameter of deformation. This show that for relativistic particle inside the area with GUP the coordinates of the rest and moving frames of reference fulfill the Lorentz transformation with a few powerful speed of light.

Keywords:*Generalized Uncertainty Principle Deformed Heisenberg algebra Minimal length Galilean transformation Lorentz transformation ,speed of light.*

INTRODUCTION :

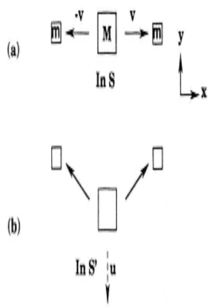
It is a typically popular rule in non-relativistic quantum concept that one can't coherently superpose particles of different masses. This rule comes from a demonstration via Bargman that, if one makes a sequence of variations to moving coordinate systems, the usage of the Galilean transformation, and ultimately arrives again at the original machine, one can have produced a segment shift among the additives of the wave function representing different mass states. but, the argument is going, those alterations are unphysical. How can merely searching at a wave characteristic from a different coordinate system probably set off a bodily that meansful segment shift that might be detected in an interference test? so as to eliminate this possibility, one imposes the super selection rule that one can't splendid pose wave capabilities of different masses. however there is a completely difficult characteristic to this result. Relativistically you may coherently integrate wave functions of various mass states, and the relevant transformation right here is the Lorentz transformation. within the non- relativistic restriction this reduces to the Galilean transformation, bu t the phase shift does not disappear in this restrict.

This show that from another point of view it's far necessary so that it will superpose special mass states non- relativistically. Then we display that there may be certainly a physical interpretation to the segment shift described above, namely that it is the "dual paradox" impact [3], the residue of the distinct proper times elapsed between the authentic inertial device, and the set of accelerating systems used to describe the particle. So those two units of coordinate structures aren't bodily equivalent. The brilliant thing is that the difference in proper times between them produces a residue inside the non-relativistic restrict. that is a segment impact and could no longer be visible in classical mechanics, however it suggests that there are non-relativistic results because of proper time in quantum theory, and a correct treatment of non-relativistic quantum theory ought to encompass the concept of proper time, and the equivalence of mass and energy. The superselection rule prohibiting the superposition of various mass states is inconsistent with the non-relativistic restrict of the Lorentz transformation.

METHODOLOGY :

Necessity of Non-Relativistic Mass Superpositions :

Consider the case of a particle of mass M at rest in an inertial system S decaying into two particles of mass m, flying off in opposite directions (along the x axis) at speed v. Non-relativistically, since mass is conserved,



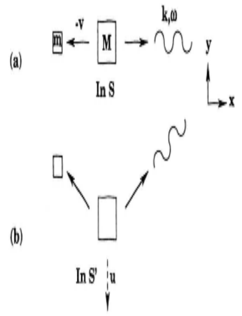


FIG 1

FIG 2

Fig. 1 Decay of a particle into identical daughter particles,(a) A particle of mass M decays into equal debris of mass m, visible from a machine S where M is at relaxation. Non-relativ- istically, because mass is conserved, M = 2m. The internal ener¬gy e components the velocity v of the daughters, and e is of order (f2/c2), as is the relativistic mass alternate, (b) The identical decay, as visible from a system S', moving alongside the - y axis at pace u.

Fig. 2 Decay of an excited particle into its ground country plus a photon, (a) The particle of mass M decays into one in all mass m plus a photon. The internal electricity e components the draw back and photon energies. right here, e and the relativistic mass change are of order v/c. (b) The equal decay, as seen from the gadget S', as before. one would have (see Figure la)

$$M = 2m, \qquad \varepsilon = mv^2. \qquad (1)$$

Here, E is the internal energy of the particle M. Non-relativistically, the mass and energy of the particle are conserved separately. Relativistically,

$$M = 2m\gamma, \quad \gamma = (1 - v^2/c^2)^{-1/2}, \quad M = 2m + \varepsilon/c^2. \qquad (2a, b, c)$$

There is no conflict here since relativistically,

$$M = 2m + O(v^2/c^2). \qquad (3)$$

Now consider the same decay from a system S', moving downward along the - y axis at velocity u (see Figure lb). Here, looking at momentum conservation (non-relativistically) along the y axis,

$$Mu = 2mu. \qquad (4)$$

This is perfectly consistent with (1). Relativistically,using

$x_\mu = (x, y, z, ict)$,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \gamma_u & 0 & -i\gamma_u u/c \\ 0 & 0 & 1 & 0 \\ 0 & i\gamma_u u/c & 0 & \gamma_u \end{bmatrix} \begin{bmatrix} v\gamma_v \\ 0 \\ 0 \\ ic\gamma_v \end{bmatrix} = \begin{bmatrix} v\gamma_v \\ u\gamma_u\gamma_v \\ 0 \\ ic\gamma_u\gamma_v \end{bmatrix}, \quad (5)$$

and momentum and energy conservation in the y direction gives

$$Mu\gamma_u = 2mu\gamma_u\gamma_v,$$
$$Mc\gamma_u = 2mc\gamma_u\gamma_v. \quad (6a, b)$$

Numerically, both of these give the same information as (2a, b, c).

but, (6a) says that what gave the impression of en¬ergy inside the rest machine S (in (2a)), looks like inertial mass in the transferring machine S', and is largely one in all Einstein's early derivations of, and is the actual meaning of, the equivalence of mass and energy. again, because of (3), there aren't any surprises right here .but, the state of affairs is very distinctive if one consid¬ers M to be the mass of an atom in an excited state, and it decays to its floor state with mass m, emitting a pho¬ton of frequency co (see figure 2a). Classically, photons do now not exist, however this is a scenario that one frequently treats non-relativistically in quantum mechanics. In this case, non-relativistic momentum conservation along the ^-course in the S gadget offers

$$mv = \hbar k.$$

In the S' system momentum conservation in the y direction gives (see Figure 2b)

$$Mu = mu + \hbar k u/c,$$

because both particle and the photon carry momentum in the y direction. But this gives

the relation

$$M = m + h\omega/c^2, \quad (9)$$

which, together with (7), gives

$$M = m(1 + v/c), \quad (10)$$

first order vie effect, in direct contrast to (3). So even non-relativistically, one cannot ignore the increase in mass, as well as in energy, of the excited state.

Relativistically, (5) gives for the photon,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \gamma_u & 0 & -i\gamma_u u/c \\ 0 & 0 & 1 & 0 \\ 0 & i\gamma_u u/c & 0 & \gamma_u \end{bmatrix} \begin{bmatrix} k \\ 0 \\ 0 \\ i\omega/c \end{bmatrix} = \begin{bmatrix} k \\ ku\gamma_u/c \\ 0 \\ i\omega\gamma_u/c \end{bmatrix} \quad (11)$$

and relativistic x-momentum and energy conservation in the system S becomes

$$mv\gamma_v = \hbar k,$$
$$Mc = mc\gamma_v + \hbar\omega/c, \quad (12)$$

while in the S' system, y-momentum and energy become

$$Mu\gamma_u = mu\gamma_u\gamma_v + \hbar k u\gamma_u/c,$$
$$Mc\gamma_u = mc\gamma_u\gamma_v + \hbar\omega\gamma_u/c. \quad (13)$$

here it is very clean that energy in the S system will become inertial mass in the S' gadget, and that the boom of mass inside the excited kingdom is important for the consisten¬cy of the principle. Non-relativistically, (12) reduces to (nine).

Quantum automatically there are many conditions where one has coherent combinations of the excited and floor country, that are normally idea of as superpo¬sitions of different electricity states. however it's miles clear that for the non-relativistic Galilean transformation to be con¬sistent with the Lorentz transformation, they ought to also be taken into consideration as superpositions of various mass states.

The Quantum-Mechanical Galilean Transformation :

The Galilean transformation represents the transformation to a system moving at constant velocity. One can get extra insight into the workings of the Galilean transformation by examining the "extended" Galilean transformation to a rigid system having an arbitrary time-de-pendent acceleration. The extended Galilean transformation [4] is given by

$$r' = r - \xi(t), \quad t' = t. \tag{14}$$

This leads to

$$\nabla = \nabla', \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - \dot{\xi}\cdot\nabla' \tag{15}$$

which, when applied to the Schrödinger equation yields

$$-\frac{\hbar^2}{2m}\nabla^2\psi = i\hbar\frac{\partial\psi}{\partial t} \to$$

$$-\frac{\hbar^2}{2m}\nabla'^2\psi = i\hbar\left(\frac{\partial}{\partial t'} - \dot{\xi}\cdot\nabla'\right)\psi. \tag{16}$$

Here we have replaced t' by t. One can eliminate the un wanted $\nabla'\psi$ term by the substitution

$$\psi(r,t) = e^{if(r',t)}\varphi(r',t). \tag{17}$$

Then,

$$\nabla'\psi = (\nabla'\psi + i\nabla'f)\, e^{if},$$
$$\nabla'^2\psi = (\nabla'^2\psi + 2i\nabla'f\cdot\nabla'\varphi + \varphi\nabla'^2 f + \varphi(\nabla'f)^2)\, e^{if},$$
$$\dot{\psi} = (\dot{\varphi} + i\dot{f}\varphi)\, e^{if}, \tag{18}$$

and the Schrödinger equation becomes

$$-\frac{\hbar^2}{2m}(\nabla'^2\varphi + 2i\nabla'f\cdot\nabla'\varphi + i\varphi\nabla'^2 f - (\nabla'f)^2\varphi)$$
$$= i\hbar\left[(\dot{\varphi} + i\dot{f}\varphi) - \dot{\xi}\cdot(\nabla'\varphi + i\varphi\nabla'f)\right]. \tag{19}$$

One can choose/such as to eliminate the terms in $\nabla'\varphi$, which gives $f = \frac{m}{\hbar}\dot{\xi}\cdot r' + g(t)$. Then one can choose g(t) such as to eliminate the purely time-dependent terms, and one finally arrives at ,

$$f = \frac{m}{\hbar}\left(\dot{\xi}\cdot r' + \frac{1}{2}\int\dot{\xi}^2\,dt\right),$$
$$-\frac{\hbar^2}{2m}\nabla'^2\varphi + m\ddot{\xi}\cdot r'\varphi = i\hbar\dot{\varphi},$$
$$\psi(r,t) = e^{i\frac{m}{\hbar}\left(\dot{\xi}\cdot r' + \frac{1}{2}\int\dot{\xi}^2 dt\right)}\varphi(r',t). \tag{20}$$

This form of the Schrödinger equation shows that in the accelerated system there appears a gravitational field, and so this is the expression of the strong equivalence principle in quantum theory. However, it can also be used to show another facet of the Galilean transfor-mation, because the phase factor has a strong physical interpretation .

Assume that there exists a superposition of two different masses, m1 and m2, so that the wave function can be written in an inertial system

$$\psi = \psi_1(m_1, r, t) + \psi_2(m_2, r, t). \tag{21}$$

Then assume that one can describe the same superposition in an accelerating system S' that obeys (14), with $\xi = \xi(t),\ \xi(0) = \xi(T) = 0$, so that the system S' performs a closed circuit and coincides with the system the S at times t = 0 and t=T, such that r' iT) = r(7). However, according to (20) one can write in S, where

$\xi(t) = 0$:

$$\psi_s(T) = \varphi_1 + \varphi_2. \qquad (22)$$

while in S':

$$\psi_{S'} = e^{i\frac{m_1}{2\hbar}\int_0^T \dot{\xi}^2 dt}\varphi_1 + e^{i\frac{m_2}{2\hbar}\int_0^T \dot{\xi}^2 dt}\varphi_2$$
$$= e^{i\frac{m_1}{2\hbar}\int_0^T \dot{\xi}^2 dt}(\varphi_1 + e^{-i\frac{\Delta m}{2\hbar}\int_0^T \dot{\xi}^2 dt}\varphi_2),$$
$$\Delta m = m_1 - m_2. \qquad (23)$$

at the time T and past, S and S' are the equal system. One has made a transformation to an accelerating system, which has lower back to the unique device at time T. therefore one has described the equal bodily machine , in one of a kind coordinate structures, and yet the second one device has brought about a relative phase shift among the 2 components relative to the primary system. This segment shift could be detectable in an interference test, but it has no bodily signifi¬cance, simply referring to how one might describe the same country in a unique coordinate device.

Bargmann brought the way round this catch 22 situation, that has been used ever when you consider that, particularly to require that Am = zero. hence, so that you can get rid of the unphysical phase shift, one places a superselection rule on the system and requires that particles of various hundreds cannot be superposed in non-relativistic quantum mechanics. (Bargmann simply achieved a sequence of translations and normal regular pace Galilean transformations, but we will display in the appendix that our subsequent argument holds in that case also.)

The Inconsistency of the Superselection Rule :

We believe that the above solution, that hundreds cannot not be superposed non-relativistically, is inconsistent with the concepts of quantum mechanics and relativity. not handiest does it contradict the result we formerly arrived at, however it does so for a totally specific cause. it is simply now not proper that the difference between the 2 systems S and S' in the preceding phase is unphysical. In truth, if one were attached to the system S' whilst it underwent its acceleration, one's clocks might be strolling at a different price than those within the device S, and while one arrived lower back in S at time T, less time would have surpassed inside the system S' than inside the system S. This impact is indeed just the same old dual paradox of unique relativity. If one closed one's eyes at time t =0, and opened them at time T, and had been asked which machine is the one that had elevated, while one should provide no solution in classical physics, this is not authentic in special relativity. There, one might say that the machine for which less time had elapsed is the machine that has been increased. and in reality, in relativistic quantum mechanics the generalization of the Galilean transformation is the Lorentz transformation, and relativistically one could superpose specific loads. The effect is actual, and it leaves a residue in the non-relativistic restriction. and actually, the difference in right instances between the 2 coordinate structures, S and S', is within the non-relativistic restriction.

$$\tau_1 - \tau_2 = t - \int\sqrt{1 - \dot{\xi}^2/c^2}\, dt \to \frac{1}{2c^2}\int \dot{\xi}^2 dt \quad (24)$$

The space-time phase factor of a plane wave to an observer moving with the particle

$$k \cdot r - \omega t = \frac{mv\gamma t}{\hbar} \cdot vt - \frac{mc^2\gamma}{\hbar} t = -\frac{mc^2\gamma}{\hbar\gamma^2} t = -\frac{mc^2\tau}{\hbar},$$

is an invariant and holds for arbitrary motion. In the systems S and S' we have

$$\begin{aligned} \psi_S &= e^{-im_1c^2\tau_1/\hbar}\varphi_1 + e^{-im_2c^2\tau_1/\hbar}\varphi_2 \\ &= e^{-im_1c^2\tau_1/\hbar}(\varphi_1 + e^{i\Delta mc^2\tau_1/\hbar}\varphi_2) \\ &= e^{-im_1c^2\tau_1/\hbar}(\varphi_1 + \varphi_2'), \\ \psi_{S'} &= e^{-im_1c^2\tau_2/\hbar}\varphi_1 + e^{-im_2c^2\tau_2/\hbar}\varphi_2 \\ &= e^{-im_1c^2\tau_2/\hbar}(\varphi_1 + e^{i\Delta mc^2\tau_2/\hbar}\varphi_2) \\ &= e^{-im_1c^2\tau_2/\hbar}(\varphi_1 + e^{-i\Delta mc^2\Delta\tau/\hbar}\varphi_2'), \end{aligned} \quad (26)$$

Where $\varphi_2' = e^{i\Delta mc^2\tau_1/\hbar}\varphi_2$, and we see that the extra phase shift in the non-relativistic limit is

$$e^{-i\Delta mc^2\Delta\tau/\hbar} \rightarrow e^{-\frac{i\Delta m}{2\hbar}\int \dot{\xi}^2 dt}, \quad (27)$$

which agrees with that of (23).

Relativistically, the phase factor $e^{-mc^2\tau/\hbar}$ is of course an invariant under a Lorentz transformation, but in deriving the non-relativistic Schrödinger equation this in-variance is destroyed by factoring out the time dependence $e^{-mc^2\tau/\hbar}$. The phase that remains, and which shows up explicitly in the Galilean transformation, is $e^{-mc^2(\tau - t)/\hbar}$ .This phase, which is independent of c, and which therefore shows up in the non-relativistic limit, is a real effect, and it leads to the phase of (23). The problem is that although real, it is uninterruptable in the non- relativistic limit, where proper time is not recognized.

the standard answer has been to dispose of this section effect by using fiat, with the advent of a superselection rule, disallowing the advent of superposition of differing hundreds. however this is inconsistent due to the fact those superpositions do occur relativistically, and that they do purpose measurable phase shifts which persist inside the non-relativistic limit. there may be no factor, or validity, in seeking to eliminate the segment shift non-relativistically, due to the fact it is actual and has a bodily interpretation .

In classical physics this form of trouble doesn't arise, as it is a segment hassle, however in quantum mechanics we've got this non-local crucial over all past instances that keeps music of the proper time difference, and it isn't legitimate to disregard it, or to legislate it away, in view that in reality it may produce real interference consequences. the appropriate way around this trouble is to concede that quantum- mechanically we need to hold music of rest-mass and proper time variations after they appear as non-vanishing levels inside the non-relativistic limit and learn how to incorporate them into the theory.

It need to no longer come as any such surprise that there are residual phase shifts because of proper time that persist inside the non-relativistic restrict, since the non-relativistic Lagrangian itself is the residuum of this type of relativistic effect. in the non-relativistic restrict we have

$$\begin{aligned} -mc^2(d\tau - dt) &= -mc^2\left(\sqrt{g_{\mu\nu}\,dx^\mu\,dx^\nu} - dt\right) \\ &\rightarrow -mc^2\left(\sqrt{g_{00} - \left(\frac{dx^i}{dt}\right)^2} - 1\right)dt \\ &\rightarrow -mc^2\left(\sqrt{1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}} - 1\right)dt \\ &\rightarrow (-mc^2 + \frac{1}{2}mv^2 - m\varphi + mc^2)\,dt \\ &\rightarrow (T - V)\,dt = L dt. \end{aligned} \quad (28)$$

These are the effects that contribute to the non-relativistic Feynman path integral, and in special circumstances can cause other quantum residual effects.

Proper Time as a Physical Variable and Operator :

One is normally used to interpreting proper time kinematically as

$$\tau = \int (1 - v^2/c^2)^{1/2}\, dt$$

along the trajectory of the particle. However this is clearly a classical inter-pretation, as it presumes a particular trajectory for the particle, an idea inconsistent with quantum mechanics. What one needs is an operator r that can be interpreted at every point (x, t) in configuration space. And just as it is only in the classical limit that x defines a trajectory, by virtue of an equation of motion, given by Hamilton's equations

$$v = \frac{dx}{dt} = \frac{\partial H}{\partial p}, \quad \frac{dp}{dt} = -\frac{\partial H}{\partial x}, \qquad (29)$$

where p is the momentum conjugate to x, so too the formalism should be able to extend itself such that the same thing happens with the proper time. It turns out that this extension is very natural [5], The mass (2 times of c) is the conjugate momentum to the proper time, and the Hamiltonian then becomes H = H (x, p,τ, m). The extra equations of motion are then

$$\frac{d\tau}{dt} = \frac{1}{c^2}\frac{\partial H}{\partial m}, \quad c^2\frac{dm}{dt} = -\frac{\partial H}{\partial \tau}.$$

As an example, for a free particle the Hamiltonian and equations of motion would be

$$\begin{aligned}
&H = \sqrt{p^2c^2 + m^2c^4} = E,\\
&v = \frac{\partial H}{\partial p} = \frac{p}{E}, \quad \frac{d\tau}{dt} = \frac{1}{c^2}\frac{\partial H}{\partial m} = \frac{mc^2}{E},\\
&\frac{dp}{dt} = \frac{dm}{dt} = 0;\\
&p = mv\Big/\sqrt{(1 - v^2/c^2)},\\
&E = mc^2\Big/\sqrt{(1 - v^2/c^2)},\\
&\frac{d\tau}{dt} = \sqrt{(1 - v^2/c^2)}, \qquad (31)
\end{aligned}$$

where the last three lines come from inverting the equation for v = v(p). Ones see here that the equation for τ(t)appears as an equation of motion and is no longer a kinematic identity. The Hamiltonian formalism also provides the operator definition of m, which is parallel to that of p, namely

$$p_{op} = \frac{\hbar}{i}\frac{\partial}{\partial x}, \quad m_{op} = \frac{1}{c^2}\frac{\hbar}{i}\frac{\partial}{\partial \tau}. \qquad (32)$$

How can one quantum-mechanically interpret r as an operator? The Galilean transformation provides the key to how this should be done. From (20) one has

$$\begin{aligned}
\psi(x,t) &= e^{im(\dot{\xi}\cdot dr' + \dot{\xi}^2 dt/2)/\hbar}\,\varphi(r', t' = \tau),\\
dr' &= dr - \dot{\xi}\, dt,\\
\psi(x,t) &= e^{im(\dot{\xi}\cdot dr - \dot{\xi}^2 dt/2)/\hbar}\,\varphi(r', \tau)\\
&= e^{[(\dot{\xi}\cdot dr - \dot{\xi}^2 dt/2)/c^2]\partial/\partial\tau}\,\varphi(r', \tau)\\
&= \varphi\left(r', \tau + \frac{1}{c^2}(\dot{\xi}\cdot dr - \dot{\xi}^2\, dt/2)\right). \qquad (33)
\end{aligned}$$

Here we have interpreted the time passed in the accelerated frame as the proper time, measured in that frame. The time tis the laboratory time. Also, we have used the fact that the mass operator acts as a translation operator in r. One then sees that this is merely the Lorentz transformation expressing itself non-relativistically,

$$d\tau = \frac{1}{\sqrt{1-\dot{\xi}^2/c^2}}(dt - \dot{\xi}\cdot dr/c^2) \xrightarrow[NR]{} dt + (\dot{\xi}^2\, dt/2 - \dot{\xi}\cdot dr)/c^2. \quad (34)$$

This yields the interpretation of τ. If the particle is located at some point (r, t1) at some time t1 ,say by passing through a slit, and a counter is placed at (r2, t2, $\dot{\xi}$,), moving with velocity $\dot{\xi}$, , then the proper time passage is given by (34), where dr = r 2- r 1 and dt = t2-t1.By altering the velocity $\dot{\xi}$, , one can alter the value of d τ, and thus one has control over the value of τ , and it can be defined at any rand τ ,and not just over a classical trajectory. One can define it along the trajectory of the particle by locating the counter on the trajectory, moving with the velocity of the particle. Thus dr = $\dot{\xi}$, dt, where $\dot{\xi}$, = v,the velocity of the particle. Similarly, if the counter is at rest, as is usually the case, then one has τ = t, and there is no information to be gained from consider-ation of the proper time. But in general one can define it along any path.

The Mass as a Physical Variable and Operator :

If one thinks of the mass as an operator, as given by(32), then the Klein-Gordon equation for a free particle in one dimension becomes

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial\tau^2} + \frac{\partial^2\psi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0. \quad (35)$$

One can change the variables to t' and u, given by

$$t' = t, \quad u = \tau - t, \quad (36)$$

and the derivatives take the form

$$\frac{\partial}{\partial\tau} = \frac{\partial}{\partial u}, \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - \frac{\partial}{\partial u}. \quad (37)$$

Then the Klein-Gordon equation becomes

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial u^2} + \frac{\partial^2\psi}{\partial x^2} - \frac{1}{c^2}\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial u}\right)^2\psi = \frac{\partial^2\psi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} + \frac{2}{c^2}\frac{\partial^2\psi}{\partial t\,\partial u} = 0. \quad (38)$$

In order to approach the non-relativistic limit, write

$$\psi = e^{i(mc^2/\hbar)u}\varphi \quad (39)$$

and drop the second time derivative term, which is down by (v power2/c power 2).Then φ obeys the non-relativistic Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\varphi}{\partial x^2} = i\hbar\frac{\partial\varphi}{\partial t}. \quad (40)$$

For a detector at rest, the solution is the standard one, as the phase factor in u disappears. But for a moving detector, the extra phase can have the same order of magnitude as the phase of the solution 1 φ. The extra phase factor in u corresponds to the $e^{-imc^2t/\hbar}$ "zitterbewegung" term factored out in the conventional solution. So in our case there is basically no relativistic zitterbewegung, but there may be a non-relativistic residue, not there in the typical principle. This answers a problem that has constantly afflicted me, namely how it's far feasible that the very unexpectedly oscillating zitterbewegung, which absolutely dominates the non-relativistic contribution, may want to perfectly cancel out, even for widely separated additives of the wave function .Relativistically, the plane wave function solution of (35) is

$$\psi = Ae^{i(mc^2\tau/\hbar + kx - \omega t)}, \qquad (41)$$

where k and ω have their relativistic values. For a detector moving along with the particle, the phase factor is equal to 0. Note that the mass in this equation does not have to be the rest mass of the particle because of the freedom given by the Klein-Gordon equation (35), where the mass is an operator. We shall see in the next section that it can contain contributions from binding energies and energy uncertainties .

Just as the phase factor in k and x gives rise to the uncertainly principle in p and x, so in the non-relativistic limit the phase factor in (39) gives rise to an uncertainty principle between u=τ- t, and m, of the form

$$c^2\Delta m \cdot \Delta u \geq \hbar/2. \qquad (42)$$

there are numerous examples of this uncertainty relation [6, 7], and it's far very widespread. If one wants to degree the mass of a particle to inside Am, then the right time on a clock sitting at the particle could be unknown to within Am .so that it will impart the taste of this relation, we are able to recall just one example. consider a charged particle, whose velocity v0is as it should be recognized, passes through a slit of width d, past that is an electric powered area perpendicular to the path of the particle (see figure three). The particle is accumulated on a screen a distance L away, and its mass is to be decided by way of its overall deflection x.

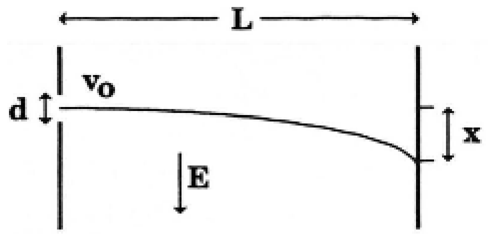


FIG 3: The mass-right time uncertainty relation: A particle of mass m, with speed v0, passes through a slit of width d and is deflected by way of an electric powered subject perpendicular to its preliminary motion. It travels a time T = L/vn, and is displaced by using a distance x. A size of x will give the value of m. A clock sitting at the particle will supply the passage of its right time τ. but the spread in transverse pace because of diffraction at the incident slit prevents an accurate degree of τ , although T is thought appropriately. This unfold in τ is correlated via v0 with the uncertainty in x, and consequently with that during m.

$$x = \frac{eE}{2m}T^2, \quad m = \frac{eE}{2x}T^2, \quad \frac{\Delta m}{m} = \frac{\Delta x}{x} \geq \frac{d}{x}. \qquad (43)$$

Here we assume that T = L/v0 can be accurately measured, and that the irreducible error in x is the size of the slit, as we cannot know where within the slit the particle passed .The passage of proper time along the path of the particle is given by

$$u = \tau - T = \frac{1}{2}\int \frac{v^2}{c^2}\, dt. \qquad (44)$$

Even if T is known very accurately, v is affected by the angular spread of the particle as it passes through the slit. We have

$$v_x = v_{0x} + \alpha t, \quad \alpha = \frac{eE}{m},$$
$$u = \frac{1}{2c^2}\int [v_0^2 + (v_{0x} + \alpha t)^2]\,dt,$$
$$\Delta u \approx \frac{1}{2c^2}\int 2\Delta v_{0x}\,\alpha t\,dt = \frac{\Delta v_{0x}\,\alpha T^2}{2c^2} = \frac{x\Delta v_{0x}}{c^2}. \quad (45)$$

Here we have used the fact that the largest error in u will be induced by the cross term in vx, since (v0x) = 0.

For a small angle of diffraction

$$\Delta v_{0x} = v_0\,\theta = v_0\,\lambda / d = v_0\,h / pd = h / md,$$
$$\Delta u = \frac{x\Delta v_{0x}}{c^2} = \frac{xh}{c^2 md},$$
$$c^2\,\Delta m \cdot \Delta u \approx c^2\left(\frac{md}{x}\right)\left(\frac{xh}{c^2\,md}\right) = h. \quad (46)$$

In this example, the m, u uncertainty relation has been reduced to the px ,x uncertainty relation, and in general this kind of phenomenon happens. So even if the time in the laboratory can be measured very accurately, the time passed on a clock moving with the particle cannot.

Quantum-Mechanical Meaning of Mass :

there may be one similarly very critical factor this is introduced out by this case, in addition to many different ex¬amples, and that is that relativistically, and quantum-routinely, the mass of a particle is not the rest mass of the loose particle. So although one knows, say, that the particle is a proton, one cannot say that Am = 0. this can be proper classically, but quantum routinely it is not. The mass is described because the strength of the particle inside the relaxation system, and quantum mechanically this generally consists of an uncertainty within the power. So the rest power of the particle may be called the "nominal" mass, but the inertial mass of the gadget, as we noticed earlier, includes these different inner energies and energy uncertainties .For a particle, or a system of debris, we are able to write

$$\bar{P} = \bar{V}\bar{E}/c^2, \quad (47)$$

where the bars indicate expectation values. The symbol V defines the average velocity of the system. One can use this equation to define a velocity operator, V0p, from the equation

$$\frac{\hbar}{i}\nabla\psi(r,t) = V_{op}\,\frac{i\hbar}{c^2}\frac{\partial\psi}{\partial t}, \quad (48)$$
$$V_{op}\,\psi = c^2\sum\frac{p_n}{E_n}a_n\,\varphi_n \xrightarrow[\text{NR}]{} c^2\sum\frac{p_n}{m_0}a_n\,\varphi_n,$$

Where $\psi = \sum a_n\,\varphi_n(r)\,e^{-iE_nt/\hbar}$,an expansion in the simultaneous functions of P and E (defined by the time derivative, and not the Hamiltonian operator.) Then the mass operator can be defined as the energy in the barycentric system, P% = 0, from a Lorentz transformation,

$$P' = P_B = \gamma_{\bar{V}}\,(P - \bar{V}\,E/c^2), \quad \langle P_B\rangle = 0,$$
$$E' = E_B = M_{op} = \gamma_{\bar{V}}\,(E - \bar{V}P), \quad \langle E_B\rangle = \bar{M},$$
$$\bar{M} = \sqrt{1 - \bar{V}^2/c^2}\;\bar{E}. \quad (49)$$

RESULTS AND DISCUSSIONS :

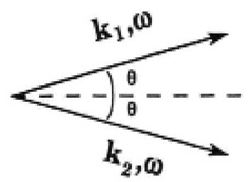


FIG 4 : The mass of a photon: A non-collinear -photon gadget has a barycentric gadget travelling with a velocity much less than c, and on this system, the electricity is the rest power of the device, and consequently its mass. further, if this were a one photon machine, with equal amplitudes for being in either of the 2 photon states, its momentum, energy expectation values will behave within the same way, and act like a large particle, despite the fact that every of its thing amplitudes has zero mass.

those equations can give sudden outcomes. For ex enough it's miles widely recognized that a -photon system, where the two photons are not collinear, has a barycentric machine. Take the 2 photons in Fig. 4, where the pholots each have the same co, but are separated through an angle 20. This device has a complete momentum P =2khkcosΘ, and energy E = 2hw. therefore the mass and velocity are, Mc^2 = 2hwsinΘnΘ, V = c cosΘ. but quantum-mechanically, for a unmarried photon, whose wave characteristic consists of a linear superposition of identical amounts of these two amplitudes, the end result is exactly the equal (without the issue of 2). that is authentic despite the fact that every separate amplitude has mass zero. this is be reason, if an ensemble of such photons struck a wall and have been absorbed, whilst every separate hit could act like a mass less particle, the expectancy fee might act similar to although the wall have been struck by using a massive particle of mass M .for this reason, in keeping with the spirit of relativity and quantum mechanics, the mass is the inertial mass of the gadget and consists of all internal energies, such as binding energies, in addition to uncertainties in the energy in the rest device , This latter is a trouble that doesn't display up classically, however the inclusion of such electricity uncertainties is vital for the consistency of the theory, and so quantum robotically one can't simply take the mass as m0, the loose particle rest mass. certainly, it'd be inconsistent to do so.

CONCLUSIONS :

we have shown that the super selection rule that masses cannot be coherently combined is inconsistent, and that there are situations in which the concepts of proper time and rest mass input within the non-relativistic restrict. This in turn clears up any other controversy that has raged almost considering the fact that the start of quantum mechanics, for plenty people have used relaxation mass and right time in non-relativistic arguments, and that they have regularly been taken to project that it become inconsistent to achieve this. possibly the most well-known case came about in Bohr's refutation of Einstein's argument that the weighing of a container of photons violates the AE At uncertainty principle. but we have visible that these principles do input non-relativistically, and in some situations are required to do so. And while it might be authentic that the injudicious use of those thoughts can motive troubles , it's also true that they do and must play a position quantum automatically in the non-relativistic limit .another factor that ought to be made is that the very idea of a wave function containing a superposition of various mass states means that the mass is the eigenprice of some operator. This paper brings an idea that has a ways-attaining implications.

REFERENCES :

[1] Daniel M. Greenberger, City College of New York, New York, NY 10031, USA(sept 17,2000), Inconsistency of the Usual Galilean Transformation in Quantum Mechanics and How To Fix It,

[2]V. Bargmann, Ann. Math. 59, 1 (1954).

[3] See also J.-M. Levy-Leblond, in Group Theory and Its Applications, Vol II, E. M. Loebl, ed., Academic Press, New York 1971; J. Math. Phys. 4, 776 (1963); and H. Bernstein, J. Math. Phys. 8, 406 (1967). It is also discussed in the text by F. A. Kaempffer, Concepts in Quantum Mechanics, Academic Press, New York 1965.

[4] D. M. Greenberger, Amer. J. Phys. 47, 35 (1979).

[5] J. Rosen, Amer. J. Phys. 40, 683 (1972). See also [3],[5] The outlines of this formalism were worked out in a series of papers: D. M. Greenberger, J. Math. Phys. 11, 2329 (1970); 11, 2341 (1970); 15, 395 (1974); 15, 406 (1974).