# A STUDY ON FOURIER TRANSFORM ANALYSIS AND ITS APPLICATIONS

**Dr.G.SUNITA SUNDARI**
HOD and Professor Department of Physics
KLEF Vaddeswaram

**Abstract -** *Fourier transformsis one of the most seasoned and an outstanding system in field of mathematic and building scientific work. Fourier change strategy speaks to the variable as a summation of complex exponentials. Fourier examination has been utilized in signal preparing and advanced picture handling for the investigation of a solitary picture as a two-dimensional wave structure, and numerous other kind of structure like Quantum mechanics, Signal handling, Image Processing. This investigation additionally speaks to channels, Transformation, portrayal, and encoding, Data Processing, Analysis and a lot more fields. The utilization of Fourier change in different applications has expanded as of late. This change is one of the least difficult change among the other change technique utilized in science. The time utilization is lesser because of this strategy. It has huge use in power circulation framework, mechanical framework, ventures and remote systems. Principally in power conveyance framework, it is the alleviation of intensity quality unsettling influence required quick, exact and high commotion invulnerable strategy. This paper has enormous of its uses of fourier change in the different fields. In wireless, the Fourier change utilizes the sign handling structures and the creation of the cell phone.*

***Key words:*** *fourier transform, signal processing, image processing, data processing, wireless networks*

**Introduction -** Fourier Transformation (FT) has enormous application in radio stargazing. Sky saw by radio telescope is recorded as the FT of genuine sky named as perceivability in radio cosmology language and this perceivability experiences Inverse Fourier Transformation and deconvolution procedure to find the genuine sky picture.

Fourier Transforms Infrared spectroscopy is a flexible device in Pharmaceutical Sciences, with a wide field of uses going from portrayal of medication definitions to explanation of motor procedures in tranquilize conveyance. Later new advancements in utilizations of these techniques incorporate investigation of medication conveyance frameworks and specifically topical drug conveyance framework. Advance methods of FTIR are Fourier Transform Infrared-Attenuated Total Reflectance (FTIR-ATR), Fourier Transform Infrared-Photo acoustic Spectroscopy (FTIR-PAS), Fourier Transform Infrared Imaging Spectroscopy, Fourier Transform Infrared Microspectrometry.

In interchanges hypothesis the sign is normally a voltage, and Fourier hypothesis is basic to seeing how a sign carries on when it goes through channels, enhancers and correspondences channels. Indeed, even discrete advanced interchanges which utilize 0's or 1's to send data still have recurrence substance. This is maybe simplest to get a handle on account of attempting to send a solitary square heartbeat down a channel.

The field of correspondences traverses a scope of uses from significant level system the board down to sending singular bits down a channel. The Fourier change is generally connected with these low level parts of correspondences.

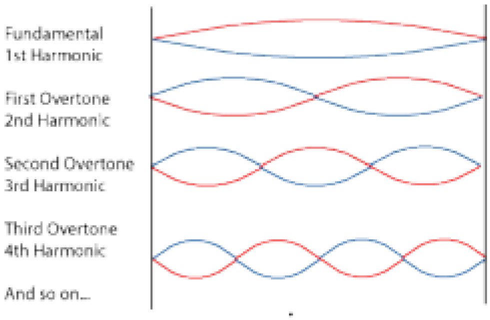
In electromagnetic hypothesis, the power of light is corresponding to the square of the swaying electric field which exists anytime in space. The Fourier change of this sign is what could be compared to breaking the light into it's segment portions of the range, a numerical spectrometer.

**Limitations-** The significant impediment of the Fourier change is the intrinsic trade off that exists among recurrence and time goals. The length of Fourier change utilized can be basic in guaranteeing that unpretentious changes in recurrence after some time, which are significant in bat echolocation calls, are seen. It might be that no single length of change is perfect for a specific sign; a few changes, every one of an alternate length, might be required before a sign can be depicted sufficiently. In a sign that is covered vigorously with clamor, the beginning and end purposes of a call might be increasingly evident in a spectrogram, in light of the fact that the high-adequacy commotion is frequently of lower otherworldly thickness and lower recurrence than the echolocation call of intrigue. Be that as it may, as a result of the windowing computation of the spectrogram, it is less fitting to gauge span of a sign from a spectrogram. Such estimations contain some level of imprecision, which can be made up for simply after expand alignment of a particular spectrogram setting (window size, window move) against the time–plentifulness show.

The improvement of the scientific build presently alluded to as the Fourier change was spurred by two issues in material science that are extremely common and perceptible. These issues are the conduction of warmth in solids and the movement of a culled string whose finishes are fixed in place.One in a split second observes the pertinence to harmony recognition since string instruments, for example, violins or pianos produce sound by intensifying the vibrations of a fixed string. The historical backdrop of the improvement of the Fourier arrangement and change is intriguing and rich.History aside, the most significant data from the advancement of Fourier investigation is that capacities can be spoken to in both the time area just as the recurrence space.

The association between the two areas is simplest to find in an occasional framework, for example, a vibrating string. One would almost certainly depict the movement of a string by concentrating on the changing situation of focuses on the string after some time. All the more thoroughly, one can demonstrate such movement utilizing a differential condition where the underlying condition is the underlying displacement.Solving such differential conditions yields a capacity f(t) (where t is time) which speaks to the movement of the string. This capacity f(t) is known as the time-space portrayal of the movement of the string; it gives data about how the string acts as time advances.

Presently envision that we pluck the string in definitely the manner in which that causes it to vibrate just at the crucial consonant below.This design is spoken to in the time area by a solitary sinusoid with recurrence ν0. Notice that the movement of the string is totally depicted by this recurrence ν0 and the plentifulness of the wavering. In this manner, the frequencydomain portrayal F(ν) of this particular string has only one spike at ν = ν0 with the tallness of the spike equivalent to the abundancy of the wave. The case of the basic symphonious pictures the recurrence space, yet in genuine frameworks there is normally more than one recurrence. To represent this, one develops the recurrence space portrayal by a vast arrangement of these sounds weighted so that they speak to the movement of the string. This arrangement is known as the Fourier arrangement and gives the premise to the Fourier change. Through the Fourier change, we can acquire the recurrence area portrayal of a period space work. The Fourier change is invertible, with the reverse Fourier change restoring the time-area work from a recurrence space work.




More rigorously, one can model such motion using a differential equation where the initial condition is the initial displacement.Solving such differential equations yields a function f(t) (where t is time) which represents the motion of the string. This function f(t) is known as the time-domain representation of the motion of the string; it provides information about how the string behaves as time progresses.

**Literary review-**

**SossioVergara, 2012:** The age of engineered signals has been one of the primary utilizations of PCs. In actuality the most punctual electronic PCs were simple and their yield was, basically, a sign. With the appearance of computerized electronic the underlying strategy utilized has been the use of the Fourier Theorem, creating signals as arrangement of sinusoids, a procedure meriting a name by its own: "added substance blend". The upside of the method is the extraordinary control on the parameters of the created wave. The fundamental detriment is the unpredictability of the calculation in question, in particular for every segment numerous examples of sinusoid should be processed, and this typically requires an extraordinary equipment to be acted progressively. The explanation is that the sine wave, in spite of the fact that being normal for physical direct frameworks, is extremely intricate in the advanced area. This article presents a compelling speculation of the polar kind of the Fourier Theorem dependent on another strategy for examination.

**Sandra Carillo, January, 2014:** Fourier arrangement are an integral asset in applied science; in fact, their significance is twofold since Fourier arrangement are utilized to uncommon present both occasional genuine capacities just as arrangements conceded by straight halfway differential conditions with allocated introductory and limit conditions. The thought rousing the acquaintance of Fourier arrangement is with rough a customary intermittent capacity, of period

T, by means of a direct superposition of trigonometric elements of a similar period T; therefore, Fourier polynomials are developed. They play, on account of standard intermittent genuine capacities, a job simple to that one of Taylor polynomials when smooth genuine capacities are considered.

**History-**" Fourier was doing scientific material science without one ruling or filling in as a descriptive word for the other," underscores Dhombres. "He started with basic tests and hypotheses as opposed to from hypothetical thinking, along these lines showing that warmth acts like a wave. Today this is a minor thought, yet one that any analyst of his time would have discovered absurd."

Fourier didn't set out from a specific speculation on the idea of warmth, yet assembled his hypothesis exclusively on realities and trials. His methodology earned him the profound respect of Auguste Comte, who believed him to be an ideal case of positivism. This vouches for the quick festival of Fourier when his hypothesis was distributed, before his eminence came and went throughout the years.

"Fourier's situation in children changed not just concerning the historical backdrop of science yet additionally with science itself," Dhombres includes. During the nineteenth century, his examination was not utilized by specialists, and didn't include in arithmetic courses at the ÉcolePolytechnique, where Fourier had all things considered educated before leaving for the Egypt crusade. Starting during the 1930s and the after war time frame, his exploration at long last returned to the bleeding edge, and was set up as a free field in the educating of investigation.

As indicated by Patrick Flandrin, senior scientist at the Physics Laboratory of the ENS in Lyonand an individual from the French Academy of Sciences, data innovation has assumed a critical job in this arrival to the spotlight. "The Fourier examination was commonplace during the 1960s, yet its utilization significantly expanded with the formation of the calculation known as the Fast Fourier Transform. It made data innovation more affordable, quicker, and progressively available."

**Methodology**

The DFT is the tested Fourier Transform and in this manner doesn't contain all frequencies shaping a picture, yet just a lot of tests which is sufficiently enormous to completely depict the spatial area picture. The amount of frequencies identifies with the amount of pixels in the spatial space picture, for instance the image in the spatial and Fourier zone are of a comparable size.

For a square image of size N×N, the two-dimensional DFT is given by:

$$F(k,l) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} f(i,j)\, e^{-\iota 2\pi \left(\frac{ki}{N} + \frac{lj}{N}\right)}$$

where *f(a,b)* is the image in the spatial domain and the exponential term is the basis function corresponding to each point *F(k,l)* in the Fourier space. The equation can be interpreted as: the value of each point *F(k,l)* is obtained by multiplying the spatial image with the corresponding base function and summing the result.

The basis functions are sine and cosine waves with increasing frequencies, *i.e. F(0,0)* represents the DC-component of the image which corresponds to the average brightness and *F(N-1,N-1)* represents the highest frequency.

In a similar way, the Fourier image can be re-transformed to the spatial domain. The inverse Fourier transform is given by:

$$f(a,b) = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} F(k,l)\, e^{\iota 2\pi(\frac{ka}{N}+}$$

Note the $\frac{1}{N^2}$ normalization term in the inverse transformation. This normalization is sometimes applied to the forward transform instead of the inverse transform, but it should not be used for both.$$

To obtain the result for the above equations, a double sum has to be calculated for each image point. However, because the Fourier Transform is *separable*, it can be written as

$$F(k,l) = \frac{1}{N} \sum_{b=0}^{N-1} P(k,b)\, e^{-\iota 2\pi \frac{lb}{N}}$$

where

$$P(k,b) = \frac{1}{N} \sum_{a=0}^{N-1} f(a,b)\, e^{-\iota 2\pi \frac{ka}{N}}$$

Utilizing these two equations, the spatial area picture is first changed into a middle picture utilizing N one-dimensional Fourier Transforms. This middle of the road picture is then changed into the last picture, again utilizing N one-dimensional Fourier Transforms. Communicating the two-dimensional Fourier Transform as far as a progression of 2N one-dimensional transforms diminishes the quantity of required calculations.

Even with these computational savings, the ordinary one-dimensional DFT has $N^2$ complexity. This can be reduced to $N\ log_2 N$ if we utilize the Fast Fourier Transform (FFT) to figure the one-dimensional DFTs. This is a noteworthy improvement, specifically for enormous pictures. There are different types of the FFT and the greater part of them confine the size of the information picture that might be changed, often to $N = 2^n$ where *n* is an integer. The mathematical details are well described in the literature.

The Fourier Transform produces a mind boggling number esteemed yield picture which can be shown with two pictures, either with the genuine and nonexistent part or with extent and stage. In picture handling, regularly just the size of the Fourier Transform is shown, as it contains the greater part of the data of the geometric structure of the spatial area image. However, on the off chance that we need to re-change the Fourier picture into the right spatial space after some preparing in the recurrence area, we should try to save both size and period of the Fourier picture.

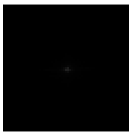
The Fourier space picture has an a lot more prominent range than the picture in the spatial area. Thus, to be adequately precise, its qualities are typically determined and put away in skim qualities.

The Fourier Transform is utilized on the off chance that we need to get to the geometric attributes of a spatial space picture. Since the picture in the Fourier space is decayed into its sinusoidal segments, it is anything but difficult to analyze or process certain frequencies of the picture, in this manner affecting the geometric structure in the spatial area.

In many usage the Fourier picture is moved so that the DC-esteem (for example the picture mean) F(0,0) is shown in the focal point of the picture. The further away from the inside a picture point is, the higher is its relating recurrence.

We start off by applying the Fourier Transform of

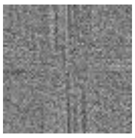
The magnitude calculated from the complex result is shown in



We can see that the DC-value is by far the largest component of the image. However, the dynamic range of the Fourier coefficients (*i.e.* the intensity values in the Fourier image) is too large to be displayed on the screen, therefore all other values appear as black. If we apply a logarithmic transformation to the image we obtain

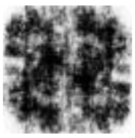
The outcome shows that the picture contains segments everything being equal, however that their greatness gets littler for higher frequencies. Subsequently, low frequencies contain more picture data than the higher ones. The change picture likewise reveals to us that there are two commanding bearings in the Fourier picture, one passing vertically and one on a level plane through the inside. These start from the ordinary examples out of sight of the first picture.

The period of the Fourier change of a similar picture is appeared in



The estimation of each point decides the period of the relating recurrence. As in the size picture, we can distinguish the vertical and level lines relating to the examples in the first picture. The stage picture doesn't yield a lot of new data about the structure of the spatial area picture; in this manner, in the accompanying models, we will confine ourselves to showing just the size of the Fourier Transform.

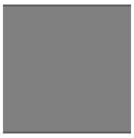
Before we leave the stage picture totally, notwithstanding, note that on the off chance that we apply the backwards Fourier Transform to the above greatness picture while overlooking the stage (and afterward histogram even out the yield) we get



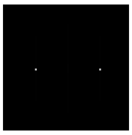
Despite the fact that this picture contains similar frequencies (and measure of frequencies) as the first info picture, it is adulterated to the point of being unrecognizable. This shows the stage data is pivotal to remake the right picture in the spatial area.

We will currently explore different avenues regarding some basic pictures to all the more likely comprehend the idea of the change. The reaction of the Fourier Transform to occasional examples in the spatial area pictures can be seen effectively in the accompanying counterfeit pictures.

The image



shows 2 pixel wide vertical stripes. The magnitude of the Fourier transform of this image is shown in



On the off chance that we look cautiously, we can see that it contains 3 principle esteems: the DC-esteem and, since the Fourier picture is even to its inside, two focuses comparing to the recurrence of the stripes in the first picture. Note that the two focuses lie on a flat line through the picture focus, in light of the fact that the picture force in the spatial area changes the most on the off chance that we come it on a level plane.

The separation of the focuses to the inside can be clarified as follows: the most extreme recurrence which can be spoken to in the spatial space are two pixel wide stripe sets (one white, one dark).

$$f_{max} = \frac{1}{2 \text{ pixels}}$$

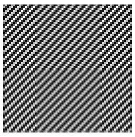
Hence, the two pixel wide stripes in the above image represent

$$f = \frac{1}{4 \text{ pixels}} = \frac{f_{max}}{2}$$

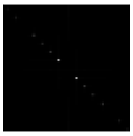
Subsequently, the focuses in the Fourier picture are somewhere between the inside and the edge of the picture, for example the spoke to recurrence is half of the greatest.

Further examination of the Fourier picture shows that the greatness of different frequencies in the picture is not exactly of the DC-esteem, for example they don't make any critical commitment to the picture. The extents of the two minor focuses are every 66% of the DC-esteem.

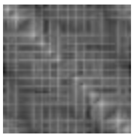
Comparative impacts as in the above model can be seen while applying the Fourier Transform to



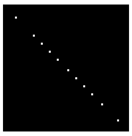
which consists of diagonal stripes. In



showing the magnitude of the Fourier Transform, we can see that, again, the main components of the transformed image are the DC-value and the two points corresponding to the frequency of the stripes. However, the logarithmic transform of the Fourier Transform,

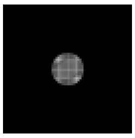


shows that now the image contains many minor frequencies. The main reason is that a diagonal can only be approximated by the square pixels of the image,hence, additional frequencies are needed to compose the image. The logarithmic scaling makes it difficult to tell the influence of single frequencies in the original image .To find the most important frequencies we threshold the original Fourier magnitude image at level 13.The resulting Fourier image. To find the most important frequencies we threshold the original Fourier magnitude image at level 13.The resulting Fourier image



shows all frequencies whose extent is at any rate 5% of the fundamental pinnacle. Contrasted with the first Fourier picture, a few additional focuses show up. They are all on a similar inclining as the three primary segments, for example they all begin from the intermittent stripes. The spoke to frequencies are for the most part products of the essential recurrence of the stripes in the spatial space picture. This is because a rectangular signal, like the stripes, with the frequency $f_{rect}$ is a composition of sine waves with the frequencies $f_{sine} = n \times f_{rect}$, known as the harmonics of $f_{rect}$. Every other recurrence vanished from the Fourier picture, for example the extent of every one of them is under 5% of the DC-esteem.

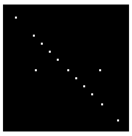
A Fourier-Transformed picture can be utilized for recurrence sifting. A straightforward model is delineated with the above picture. On the off chance that we duplicate the (mind boggling) Fourier picture got above with a picture containing a circle (of *r* = *32* pixels), we can set all frequencies larger than $f_{rect}$ to zero as shown in the logarithmic transformed image



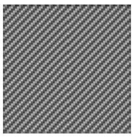
By applying the inverse Fourier Transform we obtain

The resulting image is a lowpass filtered version of the original spatial domain image. Since all other frequencies have been suppressed, this result is the sum of the constant DC-value and a sine-wave with the frequency $f_{rect}$. Further examples can be seen in the worksheet on frequency filtering.

A property of the Fourier Transform which is utilized, for instance, for the expulsion of added substance clamor, is its distributivity over option. We can delineate this by including the mind boggling Fourier pictures of the two past model pictures. To show the outcome and underscore the fundamental pinnacles, we edge the greatness of the unpredictable picture, as can be seen in

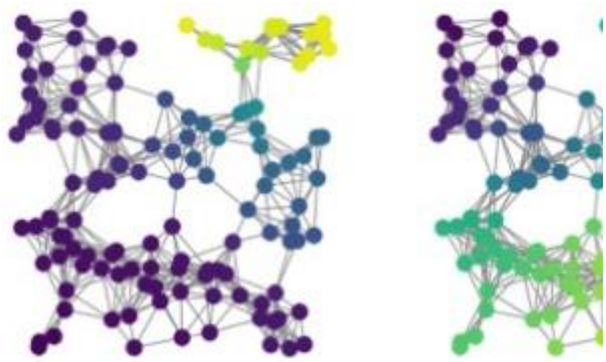


Applying the inverse Fourier Transform to the complex image yields



According to the distributivity law, this image is the same as the direct sum of the two original spatial domain images.

**Result-**



Some graph Fourier modes on a random sensor graph. From left to right: first non-constant eigenvector (Fiedler vector) $u_1$, second and third eigenvectors ($u_2$ and $u_3$). Colormap: positive values in yellow, negative ones in blue.

**Conclusions-** The proposed Fourier change has a wide scope of help in different areas like force appropriation framework, remote, signal handling, wireless assembling, mechanical and modern application. In power framework, proposed technique effectively investigates the deficiencies, music and aggravation. In remote framework, they recognize the clamor and effectively ascertaining the misfortunes in same manner. The Fourier change gives the world a simple and most agreeable strategy for arrangement of inquiries.

**References-**

*[1]SossioVergara,Published in: Digital Signal Processing Journal 22 (2012).*

*[2]Sandra Carillo, Multiphase Porous Media, High Temperature (pp.1736-1742),January 2014.*

*[3] Bucci, O.M. and Migliore, M.D. A Novel Non Uniform Fast Fourier Transform Algorithm and its Application to Aperiodic Arrays. IEEE Antennas and Wireless Propagation Letters. 2016.*

*[4] S.ShenbagaEzhil. Real Time Application of Fourier Transforms. Indonesian Journal of Electrical Engineering and Computer Science Vol. 8, No. 2, November 2017, pp. 574*

*[5] Mark E. Jakubauskas, David R. Legates, and Jude H. Kastens Harmonic Analysis of Time-Series AVHRR NDVI Data , Photogrammetric Engineering and Remote Sensing 2001*

*[6] Abhishek Jain. Recent Application of Fourier Transform In Medical Engineering. International Journal of Creative Research Thoughts (IJCRT) Volume 4, Issue 3 July 2016 pp 25*

*[7] M. Guerquin-Kern, L. Lejeune, K. P. Pruessmannet. al Realistic Analytical Phantoms for Parallel Magnetic Resonance Imaging, IEEE Transactions On Medical Imaging, Vol. 31, No. 3, March 2012*

*[8] D. Ballard and C. Brown Computer Vision, Prentice-Hall, 1982, pp 24 - 30.*

*[9]R. Gonzales, R. Woods Digital Image Processing, Addison-Wesley Publishing Company, 1992, pp 81 - 125.*

*[10]Robot Vision, MIT Press, 1986, Chaps 6, 7.*

*[11]A. Jain Fundamentals of Digital Image Processing, Prentice-Hall, 1989, pp 15 - 20.*

*[12]A. Marion An Introduction to Image Processing, Chapman and Hall, 1991, Chap. 9.*