NUMERALS FOR FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract:

Fractional differential equations are a field of mathematics study that grow out of the normal definitions of calculus integral and spin off operators in much the identical approach half way exponents is associate degree outgrowth of exponents with number price1,2,5. This paper concern with the problem that tries to approximate the solution of fractional differential equations through and how.

Key words: *Numerals, fractional differential equations.*

1. Introduction:

In recent years the isometric spline inter potation methodology as applied to the answer of differential equations employ some from approximating function such as polynomial to approximate the solution by evaluating the function for sufficient numbers of points within the domain of therefore solution six so to produce it determination of unknown Plagiarized coefficients that outline the approximating perform 3. It has been found that using spline curves, or piece-wise polynomials, is more effective in representing the solution to the differential equations [4], [8].

2. Importace of this research:

Fractional differential equations (FDEs) represent a vital tool in technology science and political economy and engineering applications enclosed population models, Management engineering electrical network analysis, gravity, medicine etc, [9],[14]. Fractional differential equation carries with it a divisional differential with given worth of the unknown perform at quite one given purpose within the domain of the answer. Recently numerical ways are used approximate at the answer of the (FDEs), that open the doors wide for future applications of those ways to robust world issues involving the numerical answer. The most common ways is cubic spline interring potation, finite distinction technique [10],[11]. This analysis can add new numerical technique (Legendre spline interpolation method) to approximate the answer of divisional differential equations.

3. Objectives of this research:

This analysis aims to the subsequent:

a).Discuss and compare the boxy spline interpolation with Legendre spline interpolation methodology.

b).Propose new method s to approximate fractional differential equations solution.

c).Use the new methodology to approximate the answer of partial aliquot differential.

d).Compare the gained leads to terms of accuracy between the boxy spline with the discuss the stability and convergent for the legendre spline interpolation method.

f).Discuss the perturbation of the answer of aliquot differential equations.

4. Research phases:

Description of the Method:

First, the aim of cubic spline interpolation is to get an interpolation formula that is continuous in both the first and second, derivatives, by spanning $\{1, x, x^2, x^3\}$, [12],[15].

with a suitable coefficients both within the some intervals and at the interpolating nodes,[15].

On the other hand, the new method (the legendre-spline interpolation) is generatored by spanning the set

$\left\{1, x, \frac{1}{2}(3x^2-1), \frac{1}{2}(5x^3-3x)\right\}$, *with a suitable coefficients, i.e*

$$L(x)=\begin{cases} a_{11}+b_{12}x+\frac{1}{2}c_{13}(3x^2-1)+\frac{1}{2}d_{14}(5x^3-3x), & x\in[a=t_1, t_2] \\ a_{i1}+b_{i2}x+\frac{1}{2}c_{i3}(3x^2-1)+\frac{1}{2}d_{i4}(5x^3-3x), & x\in[t_i, t_{i+1}] \\ a_{n1}+b_{n2}x+\frac{1}{2}c_{n3}(3x^2-1)+\frac{1}{2}d_{n4}(5x^3-3x), & x\in[t_{n-1}, t_n=b] \end{cases}$$

where the $[t_{i-1}, t_i]$ *is a regular partition on* $[a, b]$, *i.e*

$a=t_1<t_2<\ldots<t_i<t_{i+1}<\ldots<t_n=b$,

Here, $L(x)$ *satisfy the conditions,*

$$\begin{aligned} &1-L_i(t_i)=L_{i+1}(t_i), \quad i=1,\ldots\ldots,n \\ &2-L_i'(t_i)=L_{i+1}'(t_i), \quad i=1,\ldots\ldots,n \qquad \ldots\ldots\ldots\ldots (1) \\ &3-L_i''(t_i)=L_{i+1}''(t_i), \quad i=2,\ldots\ldots,n-1 \\ &4-L_1''(t_1)=0, \; L_n''(t_n)=0 \end{aligned}$$

Phase1: Background enhancement: A comprehensive of text book that covers down differential equations, their existing numerical solvers are my start to precede advanced researches.

Phase 2: analysis survey: I will use the literature survey of analysis papers that ar associated with my work, by the top of this part, i will be able to suppose:

• Write a report that cowl the prevailing solvers in my field of labor, their blessings limitations and a set of check cases and examples that covers the various classes of issues in my space of analysis.

• Write the connected existing algorithmic rule down differential equations victimisation my very own programing language.

Phase 3: focusing numerical methods: I will contemplate 2 elements on differential equation: • Initial value problems for ordinary differential equations.

• Numerals for down differential equations. Phase 4: analysis work: After the completion of the primary 3 phases, i will be able to receive the steering from my supervisor concerning what drawback in sure classes to tackle and consequently:

now, we apply the four conditions on the function $L(x)$, *to get a system of linear equations,*

$AX=B$, *where,*

• Start finding the matter victimisation different existing numerical solvers.

• Solve the issues victimization the pc algorithmic rule. A comparison is created with the numerical result given elsewhere.

Example:
The following table of values for function $f(x) = y$ is given

x_i	2	2.2	2.4	2.6
$f(x_i)$	−1.664587	−2.84835	−4.24739	−5.79257

Now, we construct the
1 − natural cubic spline,
2 − legendre − spline interpolation,

here, it is noted that $y = x^2 \cos(x)$,
Firstly: natural cubic spline:

$$A_s = \begin{bmatrix} 1 & 2 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2.2 & 4.84 & 10.64 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2.2 & 4.84 & 10.64 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2.4 & 5.76 & 13.8240 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2.4 & 5.74 & 13.824 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2.6 & 6.76 & 17.576 \\ 0 & 1 & 4.4 & 14.52 & 0 & -1 & -4.4 & -14.52 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2.8 & 17.28 & 0 & -1 & -2.8 & 17.28 \\ 0 & 0 & 2 & 13.2 & 0 & 0 & -2 & -13.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 14.4 & 0 & 0 & -2 & -14.4 \\ 0 & 0 & 2 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 15.6 \end{bmatrix}$$

$$A = \begin{bmatrix} l_{11} & l_{12} & l_{13} & l_{14} & 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ l_{21} & l_{22} & l_{23} & l_{24} & 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & l_{35} & l_{36} & l_{37} & l_{38} & 0 & 0 & \cdots & \vdots \\ 0 & 0 & 0 & 0 & l_{45} & l_{46} & l_{47} & l_{48} & 0 & 0 & \cdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & l_{ij} & l_{ij+1} & l_{ij+2} & l_{ij+3} & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & l_{i+1j} & l_{i+1j+1} & l_{i+1j+2} & l_{i+1j+3} & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & l_{n-1n-3} & l_{n-1n-2} & l_{n-1n-1} & l_{n-1n} \\ 0 & 0 & 0 & 0 & \cdots & \vdots & \vdots & \vdots & l_{nn-3} & l_{nn-2} & l_{nn-1} & l_{nn} \end{bmatrix}$$

$$X = \begin{bmatrix} a_{11} \\ b_{12} \\ c_{13} \\ d_{14} \\ \vdots \\ a_{ij} \\ b_{ij+1} \\ c_{ij+2} \\ d_{ij+3} \\ \vdots \\ a_{nn-3} \\ b_{nn-2} \\ c_{nn-1} \\ d_{nn} \end{bmatrix} \qquad B = \begin{bmatrix} y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ \vdots \\ y_{ij} \\ y_{ij+1} \\ y_{ij+2} \\ y_{ij+3} \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$X_s = \begin{bmatrix} a_{11} \\ b_{12} \\ c_{13} \\ d_{14} \\ a_{21} \\ b_{22} \\ c_{23} \\ d_{24} \\ a_{31} \\ b_{32} \\ c_{33} \\ d_{34} \end{bmatrix} \; ; \; B_s = \begin{bmatrix} -1.664587 \\ -2.84835 \\ -4.24739 \\ -5.79257 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$to\ determine\ the\ coefficients\ a_{ij}$
$we\ use\ matlap\ programme\ to\ find\ the\ inverse\ of\ A_s$

$where,\ y_{ij} = f(t_{ik}),\ i = 1, 2, \ldots, i, \ldots, k, \ldots n,$
$hence, the\ existence\ of\ unique\ solution\ depends$
$on\ l_{ij},$
$so,\ to\ solve\ this\ system\ of\ linear\ equation,\ we\ use\ a\ matlap$
$programme,$
$so,$

$$A_s^{-1} = \begin{bmatrix} -187.622 & 188.622 & 199.196 & -199.196 & -0.574 & 0.574 & -39.724 & 0.115 & -2.694 & 1.382 & 8.867 & 1.255 \\ 292.933 & -292.933 & -298.794 & 298.794 & 0.861 & -0.861 & 59.587 & -0.172 & 4.041 & -2.072 & -12.100 & -1.883 \\ -148.967 & 148.967 & 149.397 & -149.397 & -0.431 & 0.431 & -29.793 & 0.086 & -2.021 & 1.036 & 5.500 & 0.941 \\ 24.828 & -24.828 & -24.900 & 24.900 & 0.072 & -0.072 & 4.966 & -0.014 & 0.337 & -0.173 & -0.833 & -0.157 \\ 204.048 & -204.048 & -194.528 & 195.528 & 2.480 & -2.480 & 40.810 & -0.496 & -4.081 & -5.968 & 0.000 & -5.423 \\ -241.564 & 241.564 & 239.871 & -239.871 & -3.307 & 3.307 & -48.313 & 0.661 & 4.831 & 7.958 & 0.000 & 7.230 \\ 93.987 & -93.987 & -95.451 & 95.451 & 1.464 & -1.464 & 18.797 & -0.293 & -1.880 & -3.523 & 0.000 & -3.201 \\ -11.983 & 11.983 & 12.199 & -12.199 & -0.215 & 0.215 & -2.397 & 0.043 & 0.240 & 0.518 & 0.000 & 0.471 \\ 183.007 & -183.007 & -180.963 & 180.963 & 10.955 & -9.955 & 36.601 & 0.409 & -3.660 & -6.952 & 0.000 & -4.282 \\ -244.044 & 244.044 & 241.317 & -241.317 & -2.273 & 2.273 & -48.809 & -0.545 & 4.881 & 9.271 & 0.000 & 7.371 \\ 100.187 & -100.187 & -99.067 & 99.067 & -1.119 & 1.119 & 20.037 & 0.224 & -2.004 & -3.806 & 0.000 & -3.552 \\ -12.844 & 12.844 & 12.701 & -12.701 & 0.144 & -0.144 & -2.569 & -0.029 & 0.257 & 0.488 & 0.000 & 0.520 \end{bmatrix}$$

$$B_s = \begin{bmatrix} -1.664587 \\ -2.84835 \\ -4.24739 \\ -5.79257 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad now,\ compute\ A_s^{-1}B$$

$$B_s = \begin{bmatrix} -1.664587 \\ -2.84835 \\ -4.24739 \\ -5.79257 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} , \qquad now,\ compute\ A_s^{-1}B$$

$$X_s = A^{-1}B = \begin{bmatrix} 52.8480 \\ -69.9312 \\ 32.0062 \\ -5.3344 \\ -31.0234 \\ 44.5248 \\ -20.0193 \\ 2.5483 \\ -25.4014 \\ 45.2097 \\ -21.7315 \\ 2.7861 \end{bmatrix}$$

Hence; the cubic spline interpolation,

$$S(x) = \begin{cases} 52.8480 + -69.9312x + 32.0062x^2 + -5.3344x^3 , & x \in [2 \ , \ 2.2] \\ -31.0234 + 44.5248x - 20.0193x^2 + 2.5483x^3 , & x \in [2.2 \ , \ 2.4] \\ -25.4014 + 45.2097x + -21.7315x^2 + 2.7861x^3 , & x \in [2.4 \ , \ 2.6] \end{cases}$$

Secondly, we construct the legendre – spline interpolation, auxiliary matrix can be written as

$$A_L = \begin{bmatrix} 1.0000 & 2.0000 & 5.5000 & 17.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 1.0000 & 2.2000 & 6.7600 & 23.3200 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 2.2000 & 6.7600 & 23.3200 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 2.4000 & 8.1400 & 30.9600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 2.4000 & 8.1400 & 30.9600 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 2.6000 & 9.6400 & 41.5400 \\ 0.0000 & 1.0000 & 6.6000 & 34.8000 & 0.0000 & -1.0000 & -6.6000 & -34.8000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 7.2000 & 41.7000 & 0.0000 & -1.0000 & -7.2000 & -41.7000 \\ 0.0000 & 0.0000 & 3.0000 & 33.0000 & 0.0000 & 0.0000 & -3.0000 & -33.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 3.0000 & 36.0000 & 0.0000 & 0.0000 & -3.0000 & -36.0000 \\ 0.0000 & 0.0000 & 3.0000 & 30.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 3.0000 & 39.0000 \end{bmatrix}$$

$$X_L = \begin{bmatrix} a_{11} \\ b_{12} \\ c_{13} \\ d_{14} \\ a_{21} \\ b_{22} \\ c_{23} \\ d_{24} \\ a_{31} \\ b_{32} \\ c_{33} \\ d_{34} \end{bmatrix} \ ; \ B_L = \begin{bmatrix} -1.664587 \\ -2.84835 \\ -4.24739 \\ -5.79257 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

to determine the coefficients a_{ij} we use matlap programme to find the inverse of A_L ,ie,

$$A_L^{-1}=\begin{bmatrix} 53.043 & -52.043 & -32.554 & 32.554 & 10.511 & -10.511 & 8.409 & -2.102 & 0.491 & -0.140 & 1.035 & 0.070 \\ -32.204 & 32.204 & 34.005 & -34.005 & -6.801 & 6.801 & -5.441 & 1.360 & -0.317 & 0.091 & -1.239 & -0.045 \\ -1.075 & 1.075 & 1.344 & -1.344 & -0.269 & 0.269 & -0.215 & 0.054 & -0.013 & 0.004 & 0.367 & -0.002 \\ 1.075 & -1.075 & -1.344 & 1.344 & 0.269 & -0.269 & 0.215 & -0.054 & 0.013 & -0.004 & -0.034 & 0.002 \\ 519.301 & -519.301 & -1035.876 & 1036.876 & 528.575 & -528.575 & 103.860 & -105.715 & -10.040 & -7.048 & -0.346 & 3.324 \\ -580.914 & 580.914 & 1192.392 & -1192.392 & -616.478 & 616.478 & -116.183 & 123.296 & 11.231 & 8.220 & 0.387 & -4.110 \\ 158.602 & -158.602 & -335.753 & 335.753 & 177.151 & -177.151 & 31.720 & -35.430 & -3.066 & -2.362 & -0.106 & 1.181 \\ -13.441 & 13.441 & 29.301 & -29.301 & -15.860 & 15.860 & -2.688 & 3.172 & 0.260 & 0.211 & 0.009 & -0.106 \\ -134.892 & 134.892 & 795.866 & -795.866 & -647.973 & 648.973 & -26.978 & 132.195 & 2.608 & -7.914 & 0.090 & -17.846 \\ 140.054 & -140.054 & -826.317 & 826.317 & 681.263 & -681.263 & 28.011 & -137.253 & -2.708 & 8.216 & -0.093 & 19.442 \\ -34.946 & 34.946 & 206.183 & -206.183 & -171.237 & 171.237 & -6.989 & 34.247 & 0.676 & -2.050 & 0.023 & -5.142 \\ 2.688 & -2.688 & -15.860 & 15.860 & 13.172 & -13.172 & 0.538 & -2.634 & -0.052 & 0.158 & -0.002 & 0.421 \end{bmatrix}$$

Now, compute $A_L^{-1}B$ *so*

$$A_L^{-1}B=\begin{bmatrix} 2.6582 \\ -1.0563 \\ 0.1922 \\ -0.1922 \\ -22.0065 \\ 27.9701 \\ -8.2546 \\ 0.5757 \\ -53.2606 \\ 62.4143 \\ -17.5014 \\ 1.3463 \end{bmatrix}.$$

so, Legender – spline interpolation can be written as

$$L(x)=\begin{cases} 2.6582-1.0563x+\frac{0.1922}{2}(3x^2-1)-\frac{0.1922}{2}(5x^3-3x) & , x\in[2\ ,\ 2.2] \\ -22.0065+27.9701x-\frac{8.2546}{2}(3x^2-1)+\frac{0.5757}{2}(5x^3-3x) & , x\in[2.2\ ,\ 2.4] \\ -53.2606+62.41431x-\frac{17.5014}{2}(3x^2-1)+\frac{1.3463}{2}(5x^3-3x) & , x\in[2.2\ ,\ 2.4] \end{cases}$$

Here : in the following table some comparison values between Legender – spline ,Cubic spline interpolation and exact solution. see the figures (1, 2, 3) .

x_i	exact value $f(x_i)$	legendre – spline $L(x_i)$	cubic spline $S(x_i)$	$E\lvert f(x_i)-L(x_i)\rvert$	$E\lvert f(x_i)-S(x_i)\rvert$
2.123000	−2.36428	−2.366557511	−2.402791235	0.002276993	0.038510717
2.125140	−2.37727	−2.37549745	−2.415985616	0.002225264	0.038713451
2.121200	−2.35337	−2.355693127	−2.391707131	0.002319191	0.038333195
2.199900	−2.8477	−2.847702411	−2.890667291	7.22859E − 06	0.042972109
2.224000	−3.00596	−3.005419055	−2.957173168	0.000538727	0.018784614
2.300100	−3.5253	−3.524059351	−3.51400317	0.00124179	0.011297971
2.333400	−3.76127	−3.760159547	−3.753803619	0.001112135	0.007468063
2.232900	−3.06518	−3.0644856?	−3.047111778	0.000697462	0.018071354
2.399990	−4.24751	−4.247315434	−4.247273468	1.9336E − 06	4.00321E − 05
2.400010	−4.24746	−4.247464567	−3.556623177	2.4634E − 06	0.690838926
2.421100	−4.40497	−4.405592706	−3.788438933	0.000618816	0.616534957
2.499900	−5.00637	−5.009107099	−4.665091666	0.002734106	0.002734106
2.500000	−5.00715	−5.009883044	−4.6662125	0.002735447	0.340935097
2.540000	−5.31893	−5.321665416	−5.11570839	0.002738708	0.203218319
2.578990	−5.62604	−5.627451561	−5.555407452	0.001411653	0.070632476
2.599900	−5.79177	−5.791783947	−5.791497201	1.00293E − 05	0.000276717

Table (a)

Error analysis and order of convergence :

In this example we show the convergence (numarical) of the Method is good ,in addition ,the Legendre – spline method give more accuracy (Table a) than cubic spline , figures 1, 2, 3 .

Here, the Legendre – spline method produce function values $L(x)$ *as approximation to* $y(t_i)$ *.*

The unknown values may be replaced by $L(x)$ *,*

also all commands we used in appendix a, *The challenge ,is Legendre – splinemethod a good numerical approximation solution method of an initial value problem of ordinary differential equations. Fractional differential equation, partial Fractional differential equation,? What is the stability and convergence of the solution? What is a bout the perturbation of solution ? these qustions need more and more studing and researching to answer it ,and this is the our gools.*

5. Conclusion:

This research will add new numerical method (Legendre – spline, interpolation method) to approximate the solution of Fractional differential equations, and also concern with the problem that tries to approximate ate the solution of fractional differential equations through and how.

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