AN EFFICIENT MATHEMATICS OF THE STRAW FLUTE THROUGH ELECTRONIC TUNER

B PRAVEENA
Assistant Professor, Department Of Freshman,
Institute Of Aeronautical Engineering, Dundigal Hyderabad
bpraveenag@gmail.com

Abstract

It is possible to make a simple flute-like instrument using a straw. I used an electronic tuner to determine reference tones, and then located the required positions to create finger holes to form a diatonic scale. Pythagoras created Pythagorean tuning as part of his research into the relationship between scales and the length of strings. The oscillations that represent a scale and the length of the strings creating it can be represented by a geometric series, showing us the close association between music and mathematics.

***Keywords**: scales, Pythagorean tuning, GS, modular arithmetic, ET*

1. Creating Sounds

One day on an NHK television show called *Necchu Jikan* ("Time for Obsessions") I saw a man named Toru Kamiya playing a flute made from a drinking straw. He flattened the straw's mouth and cut a 5-mm slit down each side, creating two reeds that would vibrate when he blew through it. Watching Mr. Kamiya's interesting performance brought out the music lover in me, and prompted me to try my hand at creating my own straw flute.

I immediately went out and bought some straws. The straws that are commonly available are 210 mm long and 6 mm in diameter, and some of them are bendable to make drinking easier. The bending type is particularly useful when you want to connect several straws

I pressed the mouth of the straw together and used scissors to cut 5 mm slits, creating two reeds. Blowing through the thing did make a sound, but it required significant effort because they were made from polypropylene, which is stiff and has a high level of shape retention. I says that there are several things one can do to prevent this, such as holding the straw shut with a clip, lightly sanding it, slightly heating it with a lighter, or using an iron to heat the straw while you press it closed.

As to how the thing should be blown, it is impossible to make any sound with a light breath as you can with a recorder. Instead, you must hold your lips tightly shut and blow quite powerfully. When I was in junior high school I played the clarinet in the school band, so I already had the knack for producing a sound with reed instruments. I was able to produce sound by blowing the straw flute in just the same way. The sound was quite lovely, much like that of an oboe.

Once you can make sounds, it is natural to want to play a tune. I knew a melody that used only the do-re-mi of the musical scale, and opening a few holes in the straw allowed me to play it like a simple trumpet.

2.Setting Reference Tones

Encouraged by my success, I next wanted to try creating a straw flute that could play not just three notes, but the seven notes of a full scale. I used a recorder as my guide to placing the finger holes, but the result was not what I had hoped for. There are over 500 years of history behind the positioning and size of the holes on that instrument, so it is no surprise that guesswork will not suffice. I therefore challenged myself to create an instrument with more precise pitch.

The first thing to determine is the base note and the length. As shown in Table 1, there are two predominant ways of naming the notes of a scale, and most instruments use the series as shown in the table, starting and ending with “do.” The most common reference tone is A4 (“la”), however, which is 440 hertz. Hertz (abbreviated Hz) is a unit of vibrational frequency, and 440 Hz means 440 vibrations per second. Humans have a specific range of vibrations that they can hear, usually said to be from 20 to somewhere between 15,000 and 20,000 Hz. A standard 88-key piano has a range that goes up to about 4000 Hz using keys labeled A0 up to C8. The A4 key is right about at the center of the keyboard, and so is used as the reference note.

Since wind instruments begin their scales with “do,” I took C4 as a reference note. A longer instrument produces a deeper sound, and a shorter instrument a higher sound, so some tuning is required to create a perfect C4 note. In the past, U-shaped tuning forks were used to tune instruments, but these days there are electronic tuners that can be obtained inexpensively, so I bought one. Playing the straw at its current length displayed its scale, and while playing around with this clever little device I found that I could produce a C4 note with a straw 304 mm long.

Do	Re	Mi	Fa	Sol	La	Ti	Do
C	D	E	F	G	A	B	C

Table 1: The musical scale

Since store-bought straws are only 210 mm long, to lengthen one to 304 mm I had to join two of them. You can do so using cellophane tape, but using another straw with a slightly smaller diameter makes the assembly removable, which is quite convenient.

2. Scales as Geometric Series

A diatonic scale has seven notes that repeat at the octave, do-re-mi-fa-sol-la- ti-do. The transition from “mi” to “fa” and that of “ti” to “do” are considered half steps, so in all there are five whole steps and two half steps, for a total of seven. The interval of pitches from the low “do” to the high “do” is called an octave. Examining an octave as sound frequencies, the high note will be double the frequency of its low note. Actually, it is easier to consider an octave not as seven unequal steps, but rather by converting each of the whole steps into two half steps, for a total of twelve semitones

$$5 \times 2 + 2 \times 1 = 12\ (\textit{semitones}).$$

The reference note A4 is 440 Hz, so the A3 note that is one octave below it is 220 Hz. The frequency of the C4 note found between them can be calculated according to the method of equal temperament described below.

Equal temperament divides an octave into twelve equal parts. When doing so it does not use an arithmetic progression, but rather a geometric one, as follows. The A3 note's frequency is 220 Hz and that of the A4 note is 440 Hz, for a difference of 220 Hz. However. the spacing of each note is not determined by dividing this difference by twelve, but rather by multiplying the starting 220

Hz value by a certain value twelve times so that it reaches 440 Hz. Since we want one octave to double the starting frequency, we can find that number by taking the twelfth root of two.

This number is the common ratio of the geometric series. Moving from A3 to C4 we have one full step and one half step, for a total of three half steps, and cubing the constant we found above we calculate a frequency of 262 Hz

Table 2 shows a list of frequencies from C4 to D5 that I calculated using a spreadsheet.

Scale		Temperament	Frequency (Hz)	Tube length (mm)
do	C4	1.50	262	304
re	D4	1.52	294	268
mi	E4	1.66	330	237
fa	F4	1.33	349	223
sol	G4	1.50	392	197
la	A4	1.68	440	173
ti	B4	1.89	494	153
do	C5	2.00	523	143
re	D5	2.24	587	126

Table 2: Scale frequencies and tube lengths

3. End Correction

Using the tuner, I found that I could produce a C4 note with a straw length of 304 mm. I also know the frequencies that represent the C4 through D5 notes. The next step is to use that information to calculate the required hole positions for the D4 through D5 notes.

But before doing that, I did some calculations to confirm that a tube length of 304 mm would produce a frequency of 262 Hz. To do that, I used the following formula, dredged up from my memories of physics class in high school:

Speed of sound (m/s) = Frequency (beats/s) wavelength (m)

We already know what the speed of sound is. At one standard atmosphere of pressure, it will vary according to the temperature *t*, as

$341.5 \times 0.6t\ m/s$,

so at a temperature of 15 °C, the speed is 340.65 m/s. We can simplify things a little by rounding this off to 340 m/s.

For a stringed instrument, the length of the string is one half that of the wavelength it produces, because both ends of the string are fixed and so determine the position of the nodes. For a wind instrument, the length of the tube will be either 1/2 or 1/4 the wavelength, depending on if the instrument is open or close ended. Recorders and flutes are of the open-ended variety, with both ends open and determining the peaks of the sound waves produced. The clarinet is of the closed-end type, with one end (the one with the mouthpiece) closed, thus determining the position of one node and one peak.

Like the clarinet, our straw flute is a closed instrument, so the length of the tube will be 1/4 that of the sound waves it produces. A recorder will have the same length as a straw flute, about 30 cm, but the recorder will produce a scale one octave higher since it is an open instrument.

Using 340 m/s as the speed of sound, and wanting a frequency of 262 Hz to produce a scale starting at C4, I calculated a desired tube length of 325 mm.

Tube length = wavelength / 4 = speed of sound / frequency / 4 = 325 mm

According to my calculations, my tube should have been 325 mm long to produce a C4 note, but according to my electronic tuner a length of 304 mm was required, a difference of 21 mm. Wondering why, I set out to investigate. As it turns out, in instruments with a closed end, the peaks of the waveform will form slightly outside of the open end, and so the wavelength will be four times the length of the tube plus that added bit. This adjustment is called gend correction.h I assume that the air continues to vibrate for some distance linearly in the direction of the tube, even when the tube is no longer present.

References

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