OPTIMIZATION OF A GENERAL PROBLEM IN FUNCTIONAL MATHEMATICS - A REVIEW

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Abstract:

The essential feature of the dynamic-programming approach is the structuring of optimization problems into multiple stages, which are solved sequentially one stage at a time. Although each one-stage problem is solved as an ordinary optimization problem, its solution helps to define the characteristics of the next one-stage problem in the sequence. Often, the stages represent different time periods in the problem's planning horizon. For example, the problem of determining the level of inventory of a single commodity can be stated as a dynamic program. There are many cases in practical applications where the variables of optimization are not continuous. Some or all of the variables must be selected from a list of integer or discrete values.

***Keywords**: General problems in design, mathematical functions, GA.*

Introduction

The famous naturalist Charles Darwin defined natural selection or survival of the fittest in his book (Darwin, 1929) as the preservation of favourable individual differences and variations, and the destruction of those that are injurious. In nature, individuals have to adapt to their environment in order to survive in a process called evolution, in which those features that make an individual more suitable to compete are preserved when it reproduces, and those features that make it weaker are eliminated. Such features are controlled by units called genes, which form sets known as chromosomes. Over subsequent generations not only the fittest individuals survive, but also their genes which are transmitted to their descendants during the sexual recombination process, which is called crossover.

A review of the methods for discrete variable optimization was recently presented by Bremicker et al. (1990), Vanderplaats and Thanedar (1991) and Arora et al. (1994). Several algorithms for discrete optimization problems were developed, among them branch and bound method, penalty function approach, rounding–off, cutting plane, simulated annealing, genetic algorithms, neural networks, and Lagrangian relaxation methods. It is observed that some of the methods for discrete variable optimization use the structure of the problem to speed up the search for the discrete solution. This class of methods is not suitable for implementation into a general purpose application (Arora et al., 1994). The branch and bound method, simulated annealing, and genetic algorithm are the most used methods. Herein, the literature review will be focused on these methods in the following sections.

Objectives

1. To check the developments in functional mathematics with genetic algorithm based.

2. To find out the scope and need of integral mathematics and GA in present era.

Literature Review:

Various researches have proposed different methods for shape optimization. Bletzinger [21 22] proposed inverse technique for form finding of shell structures. Authors such as Bennet and Botkin [23], Rasmussen [24], Kimmich and Ramm [25], Braibant and Fleury [26] developed computer aided optimum design tools for plane stress/strain problems. A good introduction into shape optimization problems can be found in Gotsis [27], Botkin [28], Zolesio [29], Ramm, Bletzinger and Kimmich [30].

From the mathematical point of view, two representations of variables - continuous and discrete ones can be found within the shape optimization area. The first contributions to the analysis and theory of shape optimization of continuum structureswas by Zienkiewicz and Campbell

A multi-criteria optimization approach was utilized to select the optimal values of the design variables that minimize the maximum amplitude of vibration, sound intensity level and weight of rings, as well as the total cost of the structure.

Many of the researchers dealt with the structural shape optimization with different objectives and behaviour and geometric constraints and much literature is available on shape optimization. To the author's knowledge, probably no attempted is made to optimize the mathematical inverse models. Recently author has carried out such investigations. The optimum shapes of deflection free inverse shell models for different objectives are investigated for various support conditions is investigated and the examples are presented. Besides the inverse models optimum shape were found for cylindrical shells with three support conditions and prismatic shells i.e box girder sections for bridges which are straight and curved in plan

These different problems may require different solution techniques, and solutions may be of very different natures. Bendshe,Diaz and Kikuchi125 applied the technique to obtain dynamically stiff structure. These problems were treated based on the OC algorithm.

Optimization techiniques:

General formulation, that models questions in terms of

- Decision variables: values under our control, "knobs" that we can change
- Constraints: restrict the values the decision variables can take
- Possibly, objective function(s), whose value we try to optimize

"Typical" form: minimize f0 (x 1, . . . , x n) s.t. fi(x 1, . . . , x n) ≤ 0 i = 1, . . . , m The nature of variables xi and constraints fi can be quite diverse: Variables may represent individual values, or sets, or functions, . . . Constraints may be deterministic, stochastic, logical,

Linear programming (LP)

A well-known and important case, where both the objective and constraints are affine functions minimize c T x s.t. Ax ≤ b [1 5 6]

Well-developed, mature theory Algebraic and geometry aspects are well understood Many applications (e.g., flux balance analysis)

Efficiently solvable, both combinatorial (simplex) and continuous (interior-point) algorithms

Mathematical programming:

This is a synonym for finite-dimensional optimization. Its usage predates "computer programming," which actually arose from attempts at solving optimization problems on early computers. "Programming," in the sense of optimization, survives in problem classifications such as linear programming, quadratic programming, convex programming, integer programming, and so forth. [7 10 11] When treating design optimization problems of topology type emanating from other physical domains than linear elasticity, natural analogies of compliance have been used. An almost complete analogy is found in problems based on Poisson's equation, such as heat flow and Darcy flow [12]. In Stokes flow problems compliance type objectives have also been used [9, 13, 14 15, 20]. However, a surprising property of these formulations is that black and white designs appear without the need for penalizations such as that of SIMP. The effect is similar to the self-penalization property reported in, e.g., Wein et al. [16]. An optimization formulation for naturally discrete state problems such as pipe flow has also been formulated using an objective of compliance type [17]. Contrary to truss optimization problems, this formulation does not result in a convex problem. Another area where compliance-like design objectives have been used is that of maximum degradation or damage of a load-carrying structure [2, 3, 4]. However, in these problems the objective is to maximizing compliance instead of minimizing it. It also turns out that, similarly to the self-penalization property of Darcy flow problems, certain parameterizations of damage results in purely black and white solutions. The present paper gives a unified theory that covers all of these design problems from different physical domains and in the process gives a clear explanation for the self-penalization property.[18]

A vector of design variables ξ ,which belongs to a bounded, closed and convex set Δ, is introduced. The constitutive matrix is taken to depend on the design in a specified manner, i.e., $C = C(\xi)$. It is assumed that $C(\xi)$ is positive definite for all $\xi \in \Delta$. However, as indicated in Section 2.1, an extension to the more general case of a positive semi-definite $C(\xi)$ is of interest and indicated for some particular examples in the following subsections. The objective functions of the dual optimization problems now become functions of the design and we write $\Pi(x, \xi)$ and $\Pi*(y, \xi)$. As an objective for our design optimization we take the value given , i.e., our design objective function is

$\phi(\xi) = \max x \{-\Pi(x, \xi)\} = \min y \{\Pi * (y, \xi) : A T y = f\}$. (

it is shown that $\phi(\xi) = 1\ 2\ e\ T\ 0\ y(\xi) + 1\ 2\ f\ T\ x(\xi)$,

where $y(\xi)$ and $x(\xi)$ are solutions of (3) for a given design $\xi \in \Delta$. This expression for $\phi(\xi)$ is used for interpreting its physical meaning in particular cases.

By direct calculations we may also find other expressions for

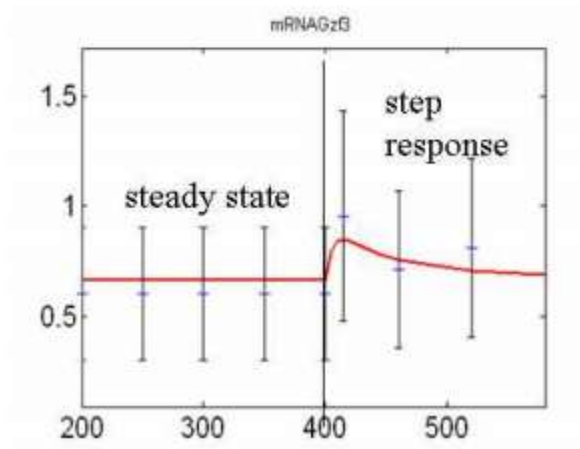
$\phi(\xi)$: $\phi(\xi) = - 1\ 2\ y(\xi)\ T\ C\ -1\ y(\xi) + f\ T\ x(\xi) = 1\ 2\ (Ax(\xi) - e0)\ T\ C(Ax(\xi) - e0) + e\ T\ 0\ y(\xi)$, which shows that if f is zero, then $\phi(\xi)$ is always non-positive and if e0 is zero, then $\phi(\xi)$ is always non-negative. Since both of the design goals of maximizing or minimizing $\phi(\xi)$ can be of interest in different situations we investigate two different design problems:

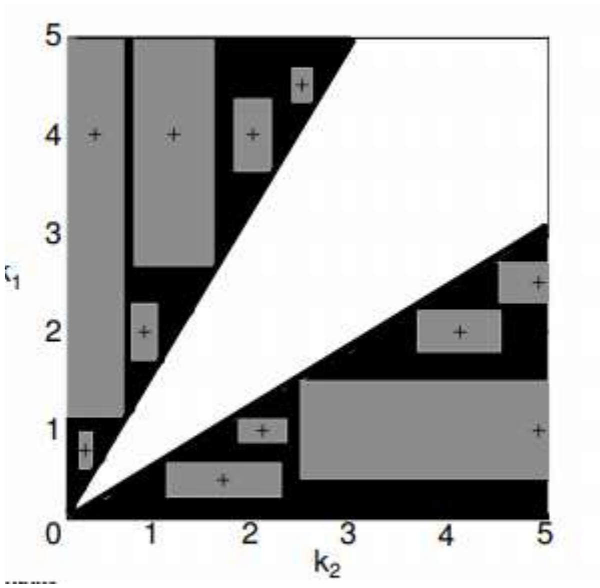
(Pmin-ϕ) min $\xi \in \Delta$ $\phi(\xi)$ and (Pmax-ϕ) max $\xi \in \Delta$ $\phi(\xi)$.

Methodological issues and caveats:

Optimization formulations are great (whenever they work), but their successful application is not always straightforward.

- An optimization approach is only as good as its formulation. Even if mathematically correct, could be nearly useless.
- Often many equivalent formulations exist, that differ substantially in efficiency and solvability. (Moderate) expertise is needed to choose appropriate descriptions.
- Optimization methods only know what they are told. All relevant features of the desired solution must be part of the model, either implicitly or explicitly





Sufficient condition for optimization

Conclusion:

Optimization techniques have many applications in bioengineering Requires careful thought at modeling time, no "black box" (yet!?) Methods have enabled many new applications Mathematical structure must be exploited for reliability and efficiency. The present paper achieves such structure by extending to the domain of design optimization the general model of linear physical theories of Tonti. By using state problem functionals in formulating objective functions, properties of convexity and concavity becomes apparent and we are given concrete guidance to which parameterizations produces a particular property.

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