THICK DOMAIN WALLS WITH BULK VISCOSITY IN EINSTEIN ROSEN CYLINDRICAL SYMMETRIC SPACE-TIME

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ABSTRACT:
In this paper, we have examined thick domain walls with bulk viscous fluid in Einstein-Rosen cylindrical symmetric space-time in the presence of cosmological constant Λ. Also, we have discussed the properties of the solutions obtained.

Keywords *- Cylindrical symmetric space-time, thick domain walls, bulk viscosity.*

INTRODUCTION

The topological defects such as domain walls, strings and monopoles have an important role in the formation of our universe. These topological defects are (often) stable configurations of matter predicted by some theories to form at phase transitions in the very early universe. Depending upon the nature of symmetry breakdown, various solutions are believed to have formed in the early universe according to the Higgs-Kibble mechanism.

Hill et al. (1989) pointed out that the formation of galaxies is due to domain walls produced during a phase transition after the time of recombination of matter and radiation. Domain walls are two-dimensional objects that form when a discrete symmetry is spontaneously broken at a phase transition. After symmetry breaking, different regions of the universe can settle into different parts of the vacuum with domain walls forming the boundaries between these regions. Domain walls have some rather peculiar properties, for example, the gravitational field of a domain wall is repulsive rather than attractive [Pradhan et al. (2005), Pradhan (2009)].

A lot of work has been done on domain walls. Pradhan (2009) evaluated general solutions for plane-symmetric thick domain walls in Lyra geometry in presence of bulk viscous fluid. Patil et al. (2010) obtained plane symmetric cosmological models of domain wall with viscous fluid coupled with electromagnetic field in general relativity by using equation of state $p = (\gamma - 1)\rho$. Yilmaz (2006) obtained Kaluza-Klien cosmological solutions for quark matter coupled to the domain wall in the context of general relativity by using anisotropy feature of the universe. Deo (2011) studied cylindrical symmetric inhomogeneous universe with thick domain walls in biometric theory of relativity. Mahanta and Biswal (2012) constructed cosmological models for domain walls coupled with quark matter in Lyra geometry. Deo (2012) studied spherically symmetric Kantowski-Sachs space-time in the context of Rosen's biometric theory with the source matter domain walls. Deo and Qureshi (2014) studied spherically symmetric space-time with the matter domain wall coupled with massive meson in the context of Rosen's biometric theory of relativity.

The effect of bulk viscosity on cosmological evolution has been investigated by a number of authors in the context of general theory of relativity and other parallel theories of gravity. Humad et al. (2016) investigated bulk viscous fluid Bianchi-I string cosmological model in general relativity by assuming the equation $\xi\theta = M$ (constant), where ξ is the coefficient of bulk viscosity and θ is the scalar of expansion. Shri Ram and Priyanka Kumari (2014) presented non-singular Bianchi type-I and V cosmological models in the presence of bulk viscous fluid and within the framework of f(R,T) gravity theory. Tiwari (2014) obtained exact solutions for anisotropic Bianchi-I model with bulk viscosity, variable gravitational constant G and cosmological constant Λ by taking vacuum density Λ proportional to Hubble parameter H and found that Λ is positive and decreasing function of time. Shri Ram et al. (2010) presented Hypersurface-homogeneous cosmological models containing a bulk viscous fluid with time varying G and Λ, where the bulk viscosity coefficient is assumed to be a power function of the energy density.

Shri Ram (2009) investigated a spatially homogeneous and anisotropic Bianchi type V model filled with an imperfect fluid with both viscosity and heat conduction within the framework of Lyra's geometry, where two different physically viable models of the universe are presented in two types of cosmologies, one with power-law expansion and other one with exponential expansion. Cosmological model with power-law expansion has an initial big-bang type singularity at $t = 0$, whereas the model with exponential expansion has a singularity in the infinite past.

Cylindrical symmetric space-time plays an important role in understanding some essential features of the universe such as formation of galaxies during early stages of their evolution. Katore et al. (2012) investigated cylindrical symmetric Einstein-Rosen cosmological model with bulk viscosity and zero-mass scalar field in Lyra geometry where the cosmological models are obtained with the help of the special law of variation for Hubble's parameter proposed by Bermann (1983). Katore et al. (2010) studied the solutions of Einstein field equations for domain walls with cosmological constant and heat flow in Einstein-Rosen cylindrical symmetric space-time when strange quark matter and normal matter attached to the domain walls and discussed some physical and kinematical features of the obtained cosmological model. Mete et al. (2016) obtained mesonic perfect fluid solutions with the aid of Einstein-Rosen cylindrical symmetric space time and investigated static vacuum model and a non-static cosmological model corresponding to perfect fluid, where the cosmological term Λ is found to be a decreasing function of time which is supported by the result found from recent type Ia Supernovae observations. Shaikh (2016) studied cylindrical symmetric Einstein-Rosen cosmological models with linear equation of state $p = a\rho + b$, where a and b are constants and discussed some physical and geometric properties of the model along with physical acceptability of the solutions.

This paper is devoted to the study of thick domain walls with bulk-viscous fluid in cylindrical symmetric Einstein-Rosen space-time in the presence of cosmological constant Λ. The paper is outlined as follows: In Section II, we have obtained Einstein field equations for thick domain wall with bulk viscous fluid in Einstein-Rosen cylindrical symmetric space-time in the presence of cosmological constant Λ. In Section III, the solutions of the field equations are obtained for thick domain wall with bulk viscous fluid. In Section IV, the properties of the solutions are discussed with concluding remarks.

1. FIELD EQUATIONS

The non-static cylindrical symmetric Einstein-Rosen metric [Mete and Elkar (2016)] is given by

$$ds^2 = e^{2(\alpha-\beta)}(dt^2 - dr^2) - r^2 e^{-2\beta} d\phi^2 - e^{2\beta} dz^2, \tag{1}$$

where α and β are functions of r and t and $x^{1,2,3,4} = r, \phi, z, t$.

Einstein's field equations in the presence of cosmological constant Λ are given by

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -8\pi\, G\, T_{ij}. \tag{2}$$

Here, we shall use geometrized units so that $8\pi G = c = 1$.

The energy momentum tensor T_{ij} for thick domain wall with bulk viscous fluid has the form [Pradhan (2009)]

$$T_{ij} = \rho\left(g_{ij} + w_i w_j\right) + \bar{p} w_i w_j, \quad w_i w^i = -1, \tag{3}$$

where

$$\bar{p} = p - \xi\, w^i_{;i} = p - \xi\theta \tag{4}$$

Here ρ, p, $\bar{p}$ and ξ are the energy density, the pressure in the direction normal to the surface of the wall, effective pressure and bulk viscous coefficient respectively, and w_i is unit space-like vector in the same direction and $\theta = w^i_{;i}$ is the scalar expansion. Using (3), the field equations (2) for the metric (1) reduce to

$$e^{-2(\alpha-\beta)}\left[\beta_1^2 + \beta_4^2 - \frac{\alpha_1}{r}\right] - \Lambda = -\bar{p} \tag{5}$$

$$e^{-2(\alpha-\beta)}\left[\alpha_{44} - \alpha_{11} - \beta_1^2 + \beta_4^2\right] - \Lambda = \rho \tag{6}$$

$$e^{-2(\alpha-\beta)}\left[2\beta_{11} - 2\beta_{44} + \frac{2\beta_1}{r} - \alpha_{11} + \alpha_{44} - \beta_1^2 + \beta_4^2\right] - \Lambda = \rho \tag{7}$$

$$e^{-2(\alpha-\beta)}\left[\beta_1^2 + \beta_4^2 - \frac{\alpha_1}{r}\right] + \Lambda = -\rho \tag{8}$$

and

$$2\beta_1\beta_4 - \frac{\alpha_4}{r} = 0, \tag{9}$$

where the suffixes 1 and 4 stand for partial derivatives w. r. t. r and t respectively.

Equations (6) and (7) gives

$$\beta_{11} - \beta_{44} - \frac{\beta_1}{r} = 0 \tag{10}$$

and equations (6) and (8) give

$$\alpha_{44} - \alpha_{11} + 2\beta_4^2 - \frac{\alpha_1}{r} = 0 \tag{11}$$

2. SOLUTIONS OF FIELD EQUATIONS

Assume $\beta = A(r) + B(t)$ and $\alpha = mA(r) + nB(t)$. (12)

Then (10) and (11) give

$$A'' - \ddot{B} + \frac{A'}{r} = 0 \text{ i.e. } A'' + \frac{A'}{r} = \ddot{B} \tag{13}$$

and

$$n\ddot{B} - mA'' + 2\dot{B}^2 - \frac{mA'}{r} = 0$$

or

$$m\left(A'' + \frac{A'}{r}\right) = n\ddot{B} + 2\dot{B}^2, \tag{14}$$

where primes and dots denote ordinary derivatives w. r. t. r and t respectively.

Equations (2) and (3) give

$$\frac{n\ddot{B} + 2\dot{B}^2}{\ddot{B}} = m \text{ or } (m-n)\ddot{B} = 2\dot{B}^2. \tag{15}$$

The general solution of (13) is given by

$$B = \frac{1}{l}\ln\left|\frac{k}{lt+q}\right|, q(>0) \tag{16}$$

where $l = \frac{2}{m-n}\,(m \neq n)$ and $q(>0), k(>0)$ are the constant of integration.

Equation (9) gives

$$2A^{'\dot{B}} - \left(\frac{n\dot{B}}{r}\right) = 0 \ \text{ or } 2A^{'} = \frac{n}{r}. \ (\because \text{ for } \dot{B} \neq 0) \tag{17}$$

The solution of equation (17) is given by

$$A = \frac{n}{2}\ln(cr), \tag{18}$$

where $c(> 0)$ is the constant of integration.
Therefore equations (12) give

$$\alpha = \frac{mn}{2}\ln(cr) + \frac{n}{l}\ln\left|\frac{k}{lt+q}\right| \tag{19}$$

and

$$\beta = \frac{n}{2}\ln(cr) + \frac{1}{l}\ln\left|\frac{k}{lt+q}\right| \tag{20}$$

Therefore $\alpha - \beta = \dfrac{n(m-1)}{2}\ln(cr) + \dfrac{(n-1)}{l}\ln\left|\dfrac{k}{lt+q}\right|$ (21)

Using (19) and (20) in (5), (6) and (8) the quantities $\bar{p}$ and ρ can be written in the form: (10)

$$\bar{p} = \Lambda - e^{-2(\alpha-\beta)}\left[\frac{n(n-2m)}{4r^2} + \frac{1}{(lt+q)^2}\right] \tag{22}$$

$$\rho = e^{-2(\alpha-\beta)}\left[\frac{(nl+1)}{(lt+q)^2} - \frac{n(n-2m)}{4r^2}\right] - \Lambda \tag{23}$$

and

$$\rho = e^{-2(\alpha-\beta)}\left[\frac{n(2m-n)}{4r^2} - \frac{1}{(lt+q)^2}\right] - \Lambda, \tag{24}$$

where $(\alpha - \beta)$ is given by equation (10).
Using (23) and (24), we get

$$nl = -2 \ \text{ or } \ n = -\frac{2}{l}.$$

and

$$l = \frac{2}{m-n} \ \text{gives } m = 0.$$

Therefore, using (19) - (24), we get

$$\alpha = \frac{2}{l^2}\ln\left|\frac{lt+q}{k}\right|, \tag{25}$$

$$\beta = -\frac{1}{l}\ln\left[(cr)\left|\frac{lt+q}{k}\right|\right] \tag{26}$$

and

$$\alpha - \beta = \frac{1}{l}\ln\left[\left(\frac{1}{cr}\right)\left|\frac{lt+q}{k}\right|^{(2-l)/l}\right]. \tag{27}$$

Using (12) and (13), we obtain

$$\bar{p} = p - \xi\theta = \Lambda - e^{-2(\alpha-\beta)}\left[\frac{1}{l^2r^2} + \frac{1}{(lt+q)^2}\right].$$

Therefore

$$p = \Lambda - e^{-2(\alpha-\beta)}\left[\frac{1}{l^2r^2} + \frac{1}{(lt+q)^2}\right] + \xi\theta \tag{28}$$

and

$$\rho = -\Lambda - e^{-2(\alpha-\beta)}\left[\frac{1}{l^2r^2} + \frac{1}{(lt+q)^2}\right], \tag{29}$$

where $(\alpha - \beta)$ is given by equation (27).
From (29), it is clear that the condition

$$\rho \geq 0 \Rightarrow \Lambda \leq -e^{-2(\alpha-\beta)}\left[\frac{1}{l^2r^2} + \frac{1}{(lt+q)^2}\right], \tag{30}$$

that is, the cosmological constant Λ should be negative.
Assume that the pressure p and density ρ satisfy the barotropic equation of state [Patil et al. (2010].

$$p = (\gamma - 1)\rho, \qquad 0 \leq \gamma \leq 2. \tag{31}$$

Then using equation (31), equations (28) and (29) give

$$\Lambda = \left(\frac{2-\gamma}{\gamma}\right)e^{-2(\alpha-\beta)}\left[\frac{1}{l^2r^2} + \frac{1}{(lt+q)^2}\right] - \frac{\xi\theta}{\gamma}. \tag{32}$$

Therefore , from (28) and (29), we obtain

$$p = \left(\frac{1-\gamma}{\gamma}\right)\left\{e^{-2(\alpha-\beta)}\left[\frac{1}{l^2r^2} + \frac{1}{(lt+q)^2}\right] - \xi\theta\right\} \tag{33}$$

and

$$\rho = \frac{1}{\gamma}\left\{\xi\theta - 2e^{-2(\alpha-\beta)}\left[\frac{1}{l^2r^2} + \frac{1}{(lt+q)^2}\right]\right\}. \tag{34}$$

The scalar expansion is given by

$$\theta = w^i_{;i} = \left(\beta_1 - \alpha_1 - \frac{1}{r}\right)e^{-(\alpha-\beta)},$$

or

$$\theta = -\left(\frac{l+1}{lr}\right)e^{-(\alpha-\beta)}, \tag{35}$$

For $\theta > 0$, we require $l \in (-1,0)$ i.e. $-1 < l < 0$.
Putting $l = -c_1$., where $0 < c_1 < 1$.
Then using (33) and (34), the quantities p and ρ are given by

$$p = \left(\frac{1-\gamma}{\gamma}\right)\left\{2e^{-2(\alpha-\beta)}\left[\frac{1}{c_1^2r^2} + \frac{1}{(q-c_1t)^2}\right] - \xi\theta\right\}, \tag{36}$$

and

$$\rho = \frac{1}{\gamma}\left\{\xi\theta - 2e^{-2(\alpha-\beta)}\left[\frac{1}{c_1^2r^2} + \frac{1}{(q-c_1t)^2}\right]\right\} \tag{37}$$

where

$$\theta = \left(\frac{1-c_1}{c_1 r}\right)e^{-(\alpha-\beta)}, \qquad 0 < c_1 < 1 \tag{38}$$

and

$$\alpha - \beta = -\frac{1}{c_1}\ln\left[\left(\frac{1}{cr}\right)\left|\frac{q-c_1t}{k}\right|^{(2+c_1)/c_1}\right]. \tag{39}$$

3. PARTICULAR CASES

In most of the investigations involving bulk viscosity, it is assumed that

$$\xi = \xi_0 \rho^n, \tag{40}$$

where ξ_0 and n are constants [Pradhan (2009)].
Here, we consider two models: (I) $\xi = \xi_0 \rho$ and (II) $\xi = \xi_0$, where ξ_0 is constant.

Model I: Let $\xi = \xi_0 \rho$.
(41)
Then equation (37) gives

$$\rho = \left(\frac{2}{\xi_0 \theta - \gamma}\right) e^{-2(\alpha-\beta)} \left[\frac{1}{c_1^2 r^2} + \frac{1}{(q - c_1 t)^2}\right] \tag{42}$$

and from (36), we get

$$\rho = \frac{-2(1-\gamma)}{(\xi_0 \theta - \gamma)} e^{-2(\alpha-\beta)} \left[\frac{1}{c_1^2 r^2} + \frac{1}{(q - c_1 t)^2}\right] \tag{43}$$

where the quantities θ and $(\alpha - \beta)$ are given by equations (36) and (39) respectively.
Here, we consider three cases : (i) $\gamma = 0$ (dust distribution), (ii) $\gamma = 1$,
(iii) $\gamma = \frac{4}{3}$ (disordered radiation) and (iv) $\gamma = 2$ (stiff fluid).
Case (i) : For $\gamma = 0$, equations (42) and (43) give

$$p = -\rho = -\frac{2}{\xi_0 \theta} e^{-2(\alpha-\beta)} \left[\frac{1}{c_1^2 r^2} + \frac{1}{(q - c_1 t)^2}\right]. \tag{44}$$

That is, $p + \rho = 0$.

Case(ii): For $\gamma = 1$, equations (42) and (43) give

$$\rho = \frac{2}{(\xi_0 \theta - 1)} e^{-2(\alpha-\beta)} \left[\frac{1}{c_1^2 r^2} + \frac{1}{(q - c_1 t)^2}\right] \tag{45}$$

and

$$p = 0. \tag{46}$$

Case (iii): For $\gamma = \frac{4}{3}$, equations (42) and (43) give

$$p = \frac{1}{3}\rho = \frac{2}{(3\xi_0 \theta - 4)} e^{-2(\alpha-\beta)} \left[\frac{1}{c_1^2 r^2} + \frac{1}{(q - c_1 t)^2}\right] \tag{47}$$

That is, $\rho = 3p$.

Case (iv) : For $\gamma = 2$, equations (42) and (43) give

$$p = \rho = \frac{2}{(\xi_0 \theta - 2)} e^{-2(\alpha-\beta)} \left[\frac{1}{c_1^2 r^2} + \frac{1}{(q - c_1 t)^2}\right] \tag{48}$$

which gives stiff fluid solution.

Model II: Let $\xi = \xi_0$(constant). Then from equation (26) and (27), we get

$$p = \left(\frac{1-\gamma}{\gamma}\right) \left\{2e^{-2(\alpha-\beta)} \left[\frac{1}{c_1^2 r^2} + \frac{1}{(q - c_1 t)^2}\right] - \xi_0 \theta\right\}, \tag{49}$$

and

$$\rho = \frac{1}{\gamma}\left\{\xi_0\theta - 2e^{-2(\alpha-\beta)}\left[\frac{1}{c_1^2 r^2} + \frac{1}{(q - c_1 t)^2}\right]\right\}, \tag{50}$$

where the quantities θ and $(\alpha - \beta)$ are given by equation (38) and (39) respectively.
Case(i): For $\gamma = 0$, from (49) and (50), we get $p \to \infty$ and $\rho \to \infty$.

Case(ii): For $\gamma = 1$, equations (49) and (50) give

$$p = 0, \tag{51}$$

and

$$\rho = \frac{1}{\gamma}\left\{\xi_0\theta - 2e^{-2(\alpha-\beta)}\left[\frac{1}{c_1^2 r^2} + \frac{1}{(q - c_1 t)^2}\right]\right\}, \tag{52}$$

Case (iii): For $\gamma = \frac{4}{3}$, equations (51) and (52) give

$$p = \frac{1}{3}\rho = \frac{1}{4}\left\{\xi_0\theta - 2\,e^{-2(\alpha-\beta)}\left[\frac{1}{c_1^2 r^2} + \frac{1}{(q - c_1 t)^2}\right]\right\}. \tag{53}$$

Case (iv): For $\gamma = 2$, equations (6.4.10) and (6.4.11) give

$$p = \rho = \frac{\xi_0\theta}{2} - e^{-2(\alpha-\beta)}\left[\frac{1}{c_1^2 r^2} + \frac{1}{(q - c_1 t)^2}\right] \tag{54}$$

4. Conclusion

In this chapter, we have studied thick domain walls with bulk viscous fluid in Einstein-Rosen cylindrical symmetric space-time in the presence of cosmological constant Λ. The physical quantities, spatial volume and the scalar expansion θ have the following expressions for line element (1):

Spatial volume is $V = \sqrt{-g} = re^{2(\alpha-\beta)}$,

that is, $V = r\left(\frac{1}{cr}\right)^{2/c_1}\left[\frac{(q - c_1 t)}{k}\right]^{2(2-c_1)/c_1^2}$.

From above expression it is clear that the spatial volume is infinite, when $t \to \infty$ and zero, when $t \to \frac{q}{c_1}$.

The line element (1) takes the form:

$$ds^2 = \left(\frac{1}{cr}\right)^{2/c_1}\left[\frac{T}{k}\right]^{2(2-c_1)/c_1^2}(dt^2 - dr^2) - \left[\frac{1}{cr^{(1-c_1)}}\right]^{2/c_1}\left[\frac{k}{T}\right]^{2/c_1} d\phi^2 - (cr)^{2/c_1}\left[\frac{T}{k}\right]^{2/c_1} dz^2,$$

where $T = |q - c_1 t|$.

From the above line element, it is clear that our model has singularity at finite past.

Scalar expansion θ is given by

$$\theta = K_1 r^{(1-c_1)/c_1} \times \left(\frac{1}{T}\right)^{2(2-c_1)/c_1^2}, \text{ where } K_1 > 0.$$

This implies that θ tends to zero as $T \to \infty$. A network of domain walls accelerates the expansion of universe. Further it is observed that the cosmological constant Λ is negative. At every instant, the domain wall density and pressure perpendicular to the surface of domain wall decreases both sides of wall away from symmetry axis as $T \rightarrow \infty$.

REFERENCES

[1] A. Pradhan, P. Verma, arXiv: gr-qc/0508087v1 20 , 2005.

[2] A. Pradhan,, Commun. Theor. Phys. (Beijing, China) 51 (2), 2009, 378.

[3] V. R. Patil, D. D. Pawar, A. G. Deshmukh, Romanian Reports in Physics, Vol. 62 (4), 2010, 722.

[4] I. Yilmaz, Gen. Rel. Grav., Vol. 38, 2006, 1397.

[5] S. D. Deo, International Journal of Mathematical Archive, 2(1), 2011,121.

[6] K. L. Mahanta, A. K. Biswal, Journal of Modern Physics, 3, 2012, 1479.

[7] S. D. Deo, International Journal of Applied Computational Science and Mathematics, 2(1), 2012, 23.

[8] S.D. Deo, A. Qureshi, Archives of Applied Science Research, 6 (4), 2014, 91.

[9] V. Humad, H. Nagar, S. Shrimali, IOSR Journal of Mathematics, 12 (2), 2016, 11.

[10] Shri Ram, Priyanka Kumari, Cent. Eur. J. Phys.,12(10), 2014,744.

[11] R.K. Tiwari, The African Review of Physics, 2014, 9:0053.

[12] Shri Ram, Verma, M. K., Astrophysics and Space Science, 330(1), 2010, 151,

[13] Shri Ram, Verma, M.,K., Mohd Zeyauddin, Modern Physics Letters A, 24(23), 2009, 1847.

[14] S. D. Katore, A.Y. Shaikh, M. M. Sancheti, J. L. Pawade, Prespacetime Journal, 3(1), 2012, 83.

[15] M. S. Berman, Nuovo Cimento, 74B,1983, 182.

[16] S. D. Katore, R. S. Rane, K. S. Wankhade, , Int. J. Theor. Phys., 49, 2010, 1929.

[17] V. G. Mete, V. D. Elkar, International Journal of Astronomy and Astrophysics, 6 (1), 2016, Article ID: 65115,6 pages

[18] A.Y. Shaikh, The African Review of Physics, 2016, 1:0002