OBLONG DIFFERENCE MEAN PRIME LABELING OF SOME TREE GRAPHS

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ABSTRACT

*The labeling of a graph, we mean assign some integers to the vertices or edges (or both) of the graph. Here the vertices of the graph are labeled with oblong numbers and the edges are labeled with mean of the absolute difference of the end vertex values. Here the greatest common incidence number (**gcin**) of a vertex of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the **gcin** of each vertex of degree greater than one is 1, then the graph admits oblong difference mean prime labeling. Here we characterize some tree graphs for oblong difference mean prime labeling.*

Keywords : *Graph labeling, oblong numbers, prime graphs, prime labeling, trees*.

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1. INTRODUCTION

In this paper we deal with graphs that are simple, finite and undirected. The symbol V and E denote the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1], [2], [3] and [4]. Some basic concepts are taken from Frank Harary [1]. In this paper we investigated the oblong difference mean prime labeling of some tree graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The the greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor of the labels of the incident edges.

Definition: 1.2 An oblong number is the product of a number with its successor, algebraically it has the form n(n+1).The oblong numbers are 2, 6, 12, 20, --------------

2. Main Results

Definition 2.1 Let G be a graph with p vertices and q edges . Define a bijection

f : V(G) → {2,6,12,20 ,------------------,p(p+1)} by f(v_i) = $i(i+1)$, for every i from 1 to p and define a 1-1 mapping f^*_{odmpl} : E(G) → set of natural numbers N by $f^*_{odmpl}(uv)$ = | $\frac{f(u)-f(v)}{2}$ |. The induced function f^*_{odmpl} is said to be an oblong difference mean prime labeling, if the ***gcin*** of each vertex of degree at least 2, is one.

Definition 2.2 A graph which admits oblong difference mean prime labeling is called an oblong difference mean prime graph.

Theorem 2.1 Let G be the graph obtained by joining pendant edges to each vertex of path P_n. G is denoted by the symbol $P_n \Theta K_1$ and is called the comb grapg. G admits oblong difference mean prime labeling.

Proof : Let $G = P_n \odot K_1$ and let v_1, v_2,----------------,v_{2n} are the vertices of G.
Here $|V(G)| = 2n$ and $|E(G)| = 2n-1$.
Define a function $f : V \rightarrow \{2,6,12,\text{---},2n(2n+1)\}$ by
$f(v_i) = i(i+1)$, i = 1,2,---,2n.
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{odmpl} is defined as follows
$f^*_{odmpl}(v_{2i-1}\, v_{2i}) = 2i$, i = 1,2,---,n
$f^*_{odmpl}(v_{2i}\, v_{2i+2}) = 4i+3$, i = 1,2,---,n-1
Clearly f^*_{odmpl} is an injection.
gcin of of (v_2) = gcd of $\{f^*_{odmpl}(v_1\, v_2), f^*_{odmpl}(v_2\, v_4)$
= gcd of { 2, 7} = 1.
gcin of of (v_{2i+2}) = gcd of $\{f^*_{odmpl}(v_{2i}\, v_{2i+2}), f^*_{odmpl}(v_{2i+1}\, v_{2i+2})\}$ =
gcd of {4i+3, 2i+2}
= gcd of {2i+1, 2i+2}
= 1, i = 1,2,---,n-1
So, gcin of each vertex of degree greater than one is 1.
Hence $P_n \odot K_1$, admits oblong difference mean prime labeling.

Theorem 2.2 Let G be the graph obtained by joining two pendant edges to each vertex of a path P_n. G is called centipede graph. G admits oblong difference mean prime labeling.

Proof: Let G be the graph and let v_1, v_2,---,v_{3n} are the vertices of G.
Here $|V(G)| = 3n$ and $|E(G)| = 3n-1$.
Define a function $f : V \rightarrow \{2,6,12,\text{---},3n(3n+1)\}$ by
$f(v_i) = i(i+1)$, i = 1,2,---,3n.
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{odmpl} is defined as follows
$f^*_{odmpl}(v_i\, v_{i+1}) =$ i+1, 1,2,---,n+1
$f^*_{odmpl}(v_{i+2}\, v_{n+i+2}) = \frac{n^2+5n+2ni}{2}$,
i = 1,2,---,n-2
$f^*_{odmpl}(v_{i+1}\, v_{2n+i}) = 2n^2+2ni+n-i-1$,
i = 1,2,---,n

Clearly f^*_{odmpl} is an injection.
gcin of (v_{i+1}) = gcd of $\{f^*_{odmpl}(v_i\, v_{i+1}), f^*_{odmpl}(v_{i+1}\, v_{i+2})\}$
= gcd of { (i+1, i+2} = 1, i = 1,2,---,n
So, gcin of each vertex of degree greater than one is 1.
Hence C(2,n) ,admits oblong difference mean prime labeling.

Theorem 2.3 Let G be the graph obtained by joining two pendant edges to each internal vertex of path P_n.G is called Twig graph. G admits oblong difference mean prime labeling.

Proof: Let G be the graph and let v_1, v_2,---,v_{3n-4} are the vertices of G.
Here $|V(G)| = 3n-4$ and $|E(G)| = 3n-5$.
Define a function $f : V \rightarrow \{2,6,12,\text{---},(3n-4)(3n-3)\}$ by
$f(v_i) = i(i+1)$, i = 1,2,---,3n-4.
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{odmpl} is defined as follows
$f^*_{odmpl}(v_i\, v_{i+1}) =$ i+1, i = 1,2,---,n-1
$f^*_{odmpl}(v_{i+1}\, v_{n+i}) = \frac{n^2+n+2ni-2i-2}{2}$,
i = 1,2,---,n-2
$f^*_{odmpl}(v_{i+1}\, v_{2n-2+i}) = 2n^2+2ni-3n-3i$,
i = 1,2,---,n-2
Clearly f^*_{odmpl} is an injection.
gcin of (v_{i+1}) = gcd of $\{f^*_{odmpl}(v_i\, v_{i+1}), f^*_{odmpl}(v_{i+1}\, v_{i+2})\}$
= gcd of { (i+1, i+2} = 1, i = 1,2,---,n
So, gcin of each vertex of degree greater than one is 1.
Hence G, admits oblong difference mean prime labeling.

Theorem 2.4 Coconut tree graph admits oblong difference mean prime labeling.

Proof: Let G = CT(m,n) and let v_1, v_2,---,v_{m+n} are the vertices of G.
Here $|V(G)| = m+n$ and $|E(G)| = m+n-1$.
Define a function $f : V \rightarrow \{2,6,12,\text{---},(m+n)(m+n+1)\}$ by
$f(v_i) = i(i+1)$, i = 1,2,---,m+n.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{odmpl} is defined as follows

$f^*_{odmpl}\ (v_i\ v_{i+1}) =$ i+1, i = 1,2,---,m

$f^*_{odmpl}\ (v_m\ v_{m+i+1}) = \frac{i^2+3i+2mi+2m+2}{2}$, i = 1,2,---,n-1

Clearly f^*_{odmpl} is an injection.

gcin of (v_{i+1}) = gcd of $\{f^*_{odmpl}\ (v_i\ v_{i+1})$, $f^*_{odmpl}\ (v_{i+1}\ v_{i+2})\}$

= gcd of { (i+1, i+2} = 1,

i = 1,2,---,m-1

So, gcin of each vertex of degree greater than one is 1.

Hence CT(m,n), admits oblong difference mean prime labeling.

Theorem 2.5 Bistar B(m,n), admits oblong difference mean prime labeling.

Proof: Let G = B(m,n) and let v_1, v_2,---,v_{m+n+2} are the vertices of G.

Here |V(G)| = m+n+2 and |E(G)| = m+n+1.

Define a function f : V → {2,6,12,---,$(m+n+2)(m+n+3)$ } by

f(u) = 2, f(v) = 6

f(x_i) = (i+2)(i+3) , i = 1,2,---,n.

f(y_i) = (n+i+2)(n+i+3), i = 1,2,---,m.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{odmpl} is defined as follows

$f^*_{odmpl}\ (v\ x_i) = \frac{i^2+5i}{2}$, i = 1,2,---,n

$f^*_{odmpl}\ (u\ y_i) = \frac{(n+i+2)(n+i+3)-2}{2}$,

i = 1,2,---,m

$f^*_{odmpl}\ (uv)$ = 2.

Clearly f^*_{odmpl} is an injection.

gcin of (u) = 1 and gcin of(v) =1

So, gcin of each vertex of degree greater than one is 1.

Hence B(m,n), admits oblong difference mean prime labeling.

Theorem 2.6 H-graph of path P_n, admits oblong difference mean prime labeling.

Proof: Let G = H(P_n) and let v_1, v_2,---,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 2n-1.

Define a function f : V → {2,6,12,---,$2n(2n+1)$ } by

f(v_i) = i(i+1) , i = 1,2,---,2n.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{odmpl} is defined as follows

$f^*_{odmpl}\ (v_i\ v_{i+1})$ = i+1, i = 1, 2, ---, n-1

$f^*_{odmpl}\ (v_{n+i}\ v_{n+i+1})$ = n+i+1, i = 1, 2,--, n-1

Case (i) n is odd

$f^*_{odmpl}\ (v_{\frac{n+1}{2}}\ v_{\frac{3+1}{2}}) = n^2+n.$

Case (ii) n is even

$f^*_{odmpl}\ (v_{\frac{n+2}{2}}\ v_{\frac{3n}{2}}) = n^2-1.$

Clearly f^*_{odmpl} is an injection.

gcin of (v_{i+1}) = 1 and gcin of(v_{n+i+1}) =1,

i = 1,2,---,n-1

So, gcin of each vertex of degree greater than one is 1.

Hence H(P_n) , admits oblong difference mean prime labeling.

Theorem 2.7 The graph $P_n \odot K_{1,m}$,admits oblong difference mean prime labeling, if n ≢ 0(mod3).

Proof: Let G = $P_n \odot K_{1,m}$ and let v_1, v_2,---,v_{2n+nm} are the vertices of G.

Here |V(G)| = 2n+nm and |E(G)| = 2n+nm-1.

Define a function f : V → {2,6,12,---,$(2n+nm)(2n+nm+1)$ } by

f(v_i) = i(i+1) , i = 1,2,---,2n+nm.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{odmpl} is defined as follows

$f^*_{odmpl}\ (v_{2i-1}\ v_{2i})$ = 2i, i = 1,2,---,n

$f^*_{odmpl}\ (v_{2i-1}\ v_{2i+1})$ = 4i+1, i = 1,2,---,n-1

$f^*_{odmpl}\ (v_{2j}\ v_{2n+i+(j-1)m}) = \frac{(2n+i+(j-1)m)(2n+i+(j-1)m+1)-2j(2j+1)}{2}$,

j = 1,2,---,n, i = 1,2,---,m

Clearly f^*_{odmpl} is an injection.

gcin of (v_{2i-1}) = gcd of $\{f^*_{odmpl}\ (v_{2i-1}\ v_{2i})\ , f^*_{odmpl}\ (v_{2i-1}\ v_{2i+1})\ \}$

= gcd of { (2i, 4i+1} = 1, i = 1,2,---,n-1

gcin of (v_{2n-1}) = gcd of $\{f^*_{odmpl}\ (v_{2n-1}\ v_{2n-3})\ , f^*_{odmpl}\ (v_{2n-1}\ v_{2n})\ \}$

= gcd of { (4n-3, 2n} = 1.

gcin of (v_{2i}) = 1,

i = 1,2,---,n.

So, gcin of each vertex of degree greater than one is 1.

Hence $P_n \odot K_{1,m}$, admits oblong difference mean prime labeling.

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