AIJREAS VOLUME 2, ISSUE 11 (2017, NOV) (ISSN-2455-6602) ONLINE
ANVESHANA'S INTERNATIONAL JOURNAL OF RESEARCH IN ENGINEERING AND APPLIED SCIENCES

HEAT TREATMENT & TRANSIENT HEAT CONDUCTION ANALYSIS ON HOT DIE STEELS

D. MANEIAH
Professor and Head of the Department of mechanical, CMR Technical Campus, Hyderabad

R. SWETHA
Assistant professor of mechanical Department, CMR Technical Campus, Hyderabad

PILLI APPA RAO
M. Tech Student of Thermal Engineering, CMR Technical Campus, Hyderabad

ABSTRACT: *This project is carried for the heat treatment of steels in low pressure vacuum furnaces. It is a latest process which increase surface hardness, wear and corrosion resistance, anti-galling properties and improved fatigue strength. The objective of this lecture is to explain the effect of thermo-chemical transfer in the solid metals & transformation of structures. This work is also dealing with the analytical solution of unsteady state one-dimensional heat conduction problems. An improved lumped parameter model has been adopted to predict the variation of temperature field in a square bar, solid cylinder, hollow cylinder and plate. Polynomial approximation method is used to solve the transient heat conduction equations for these profiles including with boundary heat flux has been analyzed. Based on the analysis, a modified Biot number has been proposed that predicts the temperature variation irrespective of the geometry.*

Keywords – *lumped model, polynomial approximation method, transient, conduction, modified Biot number*

I INTRODCTION

In this, Steel (Hot Die Steel - Grade: H13) is carried for the heat treatment of steels in low pressure vacuum furnaces. It is a latest process which increase surface hardness, wear and corrosion resistance, anti-galling properties and improved fatigue strength. The objective of it is to explain the effect of thermo-chemical transfer in the solid metals & transformation of structures.

This work is also dealing with the analytical solution of unsteady state one-dimensional heat conduction problems. An improved lumped parameter model has been adopted to predict the variation of temperature field in a square bar, solid cylinder, hollow cylinder and plate. Polynomial approximation method is used to solve the transient conduction equations for these profiles. The obtained results are analyzed by using Heat Treatment book and Ansys Software.

II LITERATURE SURVEY

Heat Treatment and Heat Transfer are increasingly important in various areas, namely in the earth sciences, and in many other evolving areas of thermal analysis. The importance of it leads to analyze the temperature field by employing sophisticated mathematical and advanced numerical tools. It considers the various solution methodologies used to obtain the temperature field. The objective of heat treatment and heat conduction analysis are to determine the temperature field in a body and how the temperature within the portion of the body. The temperature field is usually depending on boundary conditions, initial condition, material properties and geometry of the body.

To compute the heat flux at any location, compute thermal stress, expansion, deflection, design insulation thickness, heat treatment method, these all analysis leads to know the temperature field. The solution of conduction problems involves the functional dependence of temperature on space and time coordinate. Obtaining a solution means determining the temperature distribution which is consistent with the conditions on the boundaries and also consistent with any specified constraints internal to the region.

III STRUCTURE AND OBJECTIVE OF WORK

Heat treatment of steels (Hot Die Steel - Grade: H13) in low pressure vacuum furnaces is a latest process which increase surface hardness, wear and corrosion resistance, anti-galling properties and improved fatigue strength. The main objective is to explain the effect of thermo-chemical transfer in the solid metals & transformation of structures. Heat Treatment and Heat Transfer are increasingly important in various areas, namely in the earth sciences, and in many other evolving areas of thermal analysis. It considers the various solution methodologies used to obtain the temperature field. The objective of heat treatment and heat conduction analysis are to determine the temperature field in a body and how the temperature within the portion of the body.

This work is also dealing with the analytical solution of unsteady state one-dimensional heat conduction problems. An improved lumped parameter model has been adopted to predict the variation of temperature field in a square bar, solid cylinder, hollow cylinder and plate. Polynomial approximation method is used to solve the transient conduction equations for these profiles. The obtained results are analyzed by using Heat Treatment book and Ansys Software.

S. No.	Type of Work	Software/Equipment/Source	Working Time	Remarks
1	Material Selection	Industry Standards	10 Days	-----
2	Chemical Analysis	Spectroscopy	5 Days	At Industry Lab
3	Heat Treatment	Furnaces (Hardening & Tempering)	15 Days	At Industry
4	Heat Transfer Analysis	Reference Books	10 Days	Methods – Solution
		ANSYS (Workbench-14.5) - Software		
		3D Model Design	5 Days	-----
		Thermal Analysis – Heat flux	5 Days	Transient Conduction
		Documentation	5 Days	-----
5	Results	Heat Treatment & Heat Transfer	5 Days	-----
Total duration of time			**60 Days**	

TABLE 1- WORK PROCESSING SHEET

IV MATERIAL SELECTION & CHEMICAL ANALYSIS

4.1 Hot Die Steels (Grade: H-13):

HDS Steel (H-13) is a 5% Chromium Hot Work 'Tool Steel' with higher Vanadium. It is an air hardening tool steel which combines a very good red-hardness with toughness and covers a wide variety of applications. H-13 hot work tool steel is water cooled in service and gives high tensile strength and wear-resistance. H-13 sample piece is first analyzed by using Spectroscopy Analysis for Chemical composition. And then heat treated in Hardening and Tempering processes.

Fig: 1 - Hot Die Steels

4.2 Spectroscopy (Chemical) Analysis:

Specification: SA681 GRADE: H13

Spectroscopy Analysis:

Electromagnetic radiation was the first source of energy used for spectroscopic studies. Techniques that employ electromagnetic radiations are typically classified by the wavelength region of the spectrum and include microwave, terahertz, infrared, near infrared, visible and ultraviolet, x-ray and gamma spectroscopy.

ELEMENT	PERCENTAGE
Carbon	0.4
Silicon	0.86
Manganese	0.5
Phosphorus	0.013
Sulphur	0.01
Chromium	5.18
Molybdenum	1.18
Vanadium	0.83

F

Fig: 2 - Chemical Composition

V HEAT TREATMENT

5.1 Heat Treatment of Steels (HDS Steels):

Heat treatment is an operation or aggregate of operations concerning heating at a selected

fee, soaking at a temperature for a period of time and cooling at some certain rate. The intention is to gain a desired microstructure to obtain certain predetermined houses (bodily, mechanical, magnetic or electric).
A warmness transfer method includes three stages

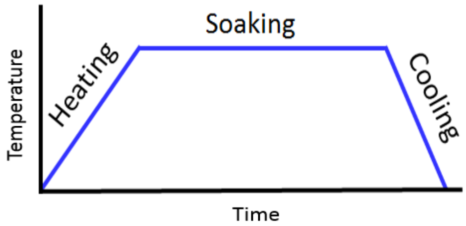


Fig: 2 - Heat Treatment of Steels

Heating: Steel is heated to pre-determined temperature above upper critical temperature [austenitizing temperature].
Socking: Steel is held in specific time to obtain homogeneous throughout the cross section.
Cooling: Heat treatment by which the structure and steel from austenite structure is cooled to room temperature depends upon the properties required stages the properties of steel can be changed.

5.2 Types of Heat Treatment Furnaces:

Conventional form of heat treatment furnaces:

- Sealed quench furnace
- Atmospheric furnace
- Salt bath furnace
- Pit furnace

Advanced form of warmness remedy furnaces:

- High vacuum furnace
- Vacuum brazing furnace
- Vacuum hardening furnace
- Vacuum tempering furnace
- Low strain vacuum nitriding furnace
- Vacuum heating oil quenching furnace

5.2.1 Vacuum Hardening Furnace:

It is having all the national and international quality standards and norms and is durable in nature and sturdily constructed. This is commonly used for heat treatment of steel alloys and involves heating the steel to a pre-determined temperature, then cooling it rapidly.

5.2.2 Vacuum Tempering Furnace:

High vacuum tempering furnace is mainly used for vacuum tempering of materials like tool steel, die steel, high-speed steel, ultra-strength steel, titanium alloy after vacuum quenching and solution treatment. It is also for re-crystallization annealing and vacuum aging of non-ferrous metal.

Fig: 3 - Vacuum Hardening Furnace

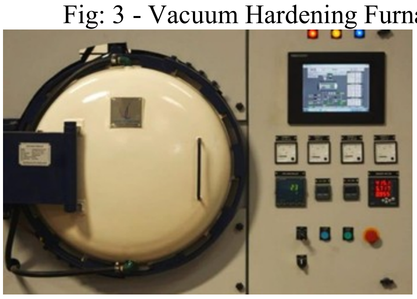


Fig: 4 - Vacuum Tempering Furnace

5.3 Areas of Applications:

1. Pressure Die-casting Dies
2. Plastic Injection Moulds
3. Plastic Compression Moulds
4. Cutting Tools
5. Hot & Warm Forging Dies
6. Hot & Cold Extrusion Dies
7. Lamination Dies
8. Surface Hardening & Stress Relieving
9. Blanking, Forming, Stamping, Progressive Dies
10. Hardening & tempering of sintered components

VI HEAT TRANSFER ANALYSIS

6.1 Heat Transfer:

Heat transfer is the study of thermal energy transport within a medium or among neighboring media by molecular interaction, fluid motion, and electro-magnetic waves, resulting from a spatial variation in temperature. This variation in temperature is governed by the principle of energy conservation, which when applied to a control volume or a control mass, states that the sum of the flow of energy and heat across the system, the work done on the system, and the energy stored and converted within the system, is zero. Heat transfer generally takes place by three modes such as conduction, convection and radiation. Heat transfer finds application in many important areas, namely design of thermal and nuclear power plants including heat engines, steam generators, condensers and other heat exchangers.

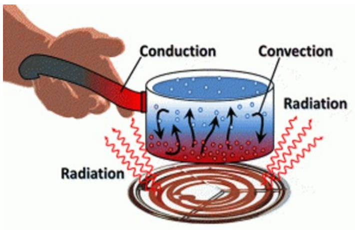


Fig: 6 - Heat Transfer

6.2 Heat Conduction Problems:
The answer of the warmth conduction problems involves the purposeful dependence of temperature on various parameters inclusive of space and time. Obtaining an answer method determining a temperature distribution which is regular with conditions on the boundaries.

6.2.1 Heat Conduction Analysis:
Heat conduction is very vital in the modern-day technology, inside the earth sciences and many other evolving regions of thermal analysis. The specification of temperatures, heat assets, and heat flux inside the areas of fabric wherein conduction occur supply upward thrust to analysis of warmth flows, temperature distribution and condition of thermal stressing. The significance of such conditions has caused a growing developed subject of analysis wherein sophisticated mathematical and numerical techniques are used. Generally, 1-dimensional, 2-dimensional and 3-dimensional coordinate machines are considered in warmness conduction. In one dimensional geometry, the temperature version in the region is described by using one variable. A plane plate and stable cylinder are considered one dimensional heat conduction when one of the surfaces of these geometries in each course is very large in comparison with the thickness.

6.2.2 One Dimensional Analysis:
In popular, the glide of warmth takes place in distinct spatial coordinates. In some instances, the evaluation is carried out via thinking about the version of temperature in one-measurement. In a plate one dimension is considered when face dimensions in every route alongside the floor are very huge compared to the vicinity thickness, with uniform boundary circumstance is carried out to every floor. Cylindrical geometries of one-measurement have axial length very huge as compared to the maximum conduction area radius. At a round geometry to have one-dimensional evaluation a uniform circumstance is carried out to each concentric floor which bounds the location.

6.2.3 Steady state and Unsteady State Analysis:
Steady state Analysis:
A constant-kingdom thermal evaluation predicts the results of regular thermal hundreds on a machine. A gadget is said to achieve constant nation while variant of diverse parameters particularly, temperature, pressure and density vary with time. A regular-state evaluation also can be taken into consideration the final step of a temporary thermal analysis. We can use constant-nation thermal evaluation to decide temperatures, thermal gradients, heat drift and heat fluxes in an item which do no longer vary over time. A consistent-kingdom thermal evaluation may be either linear, via assuming steady fabric houses or may be nonlinear case, with fabric homes varying with temperature. The thermal properties of the material do vary with temperature, so the evaluation becomes nonlinear. Furthermore, by means of considering radiation outcomes system also comes to be nonlinear.

Unsteady state Analysis:
To this point, we have got considered conductive warmness transfer problems in which the temperatures are impartial of time.

In many applications, however, the temperatures are varying with time, and we require the understanding of the entire time history of the temperature version. For example, in metallurgy, the warmth treating technique can be managed to immediately affect the traits of the processed materials. Annealing (slow cool) can melt metals and enhance ductility. On the alternative hand, quenching (fast cool) can harden the stress boundary and growth electricity.

In order to characterize this brief conduct, the overall unsteady equation is needed:

$$\frac{1}{\alpha}\frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} \quad (2.1)$$

Where, α = k/ρc is the thermal diffusivity. Without any heat era and thinking about spatial version of temperature only in x-direction, the above equation reduces to:

$$\frac{1}{\alpha}\frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial x^2} \quad (2.2)$$

For the answer of equation 2.2, we need boundary conditions in x-route and one initial circumstance. Boundary conditions, as the name implies, are frequently detailed alongside the bodily boundary of an object; they can, but, also be internal – e.g. A regarded temperature gradient at an inner line of symmetry.

6.3 Biot and Fourier numbers:

In some brief problems, the inner temperature gradients within the body can be pretty small and insignificant. Yet the temperature at a given place, or the average temperature of the item, can be converting quite unexpectedly with time. From eq. 2.1 we are able to be aware that such might be the case for big thermal diffusivity α.

A greater significant approach is to don't forget the general problem of temporary cooling of an object, inclusive of the hollow cylinder shown in figure 2.1.

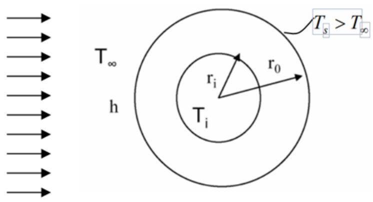


Fig: 7 - Transient Cooling of an Object

For very huge ri, the heat switch charge by conduction via the cylinder wall is about

$$q \approx -k(2\pi r_0 l)\frac{(T_s - T_i)}{(r_0 - r_i)} = k(2\pi r_0 l)\frac{(T_i - T_s)}{L} \quad (2.3)$$

Where l is the period of the cylinder and L is the fabric thickness. The fee of heat transfer far away from the outer floor by convection is

$$q = h(2\pi r_0 l)(T_s - T_\infty) \quad (2.4)$$

Where, h is the common warmness switch coefficient for convection from the whole floor. Equating (2.3) and (2.4) gives

$$\text{Biot number} = \frac{(T_i - T_s)}{(T_s - T_\infty)} = \frac{hL}{k}$$

The Biot quantity is dimensionless, and it may be thought of as the ratio.

Bi = Resistance to internal flow / Resistance to external flow

Whenever the Biot number is small, the inner temperature gradients are also small and a transient hassle may be treated via the "lumped thermal potential" approach. The lumped potential assumption implies that the item for analysis is considered to have a single mass- averaged temperature.

In the derivation shown above, the extensive item measurement became the conduction path duration, L = r0 – ri. In general, a characteristic period scale may be obtained by dividing the volume of the solid with the aid of its floor location:

$$L = V / A_s \quad (2.6)$$

Using this method to decide the characteristic length scale, the corresponding Biot wide variety can be evaluated for objects of any shape, as an example a plate, a cylinder, or a sphere. As a thumb rule, if the Biot quantity seems to be much less than zero.1, lumped ability assumption is applied. In this context, a dimensionless time, referred to as the Fourier range, is obtained by multiplying the dimensional time through the thermal diffusivity and dividing with the aid of the square of the function duration.

Dimensionless Time = $\alpha t / L^2 = F_0$

(2.7)

6. 4 Lumped Thermal Capacity Analysis:
The best state of affairs in an unsteady warmth switch method is to use the lumped capability assumption, wherein we overlook the temperature distribution within the solid and handiest cope with the heat transfer between the strong and the ambient fluids. In different phrases, we're assuming that the temperature within the solid is regular and is identical to the floor temperature.

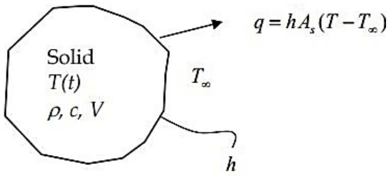


Fig: 8 - Lumped Thermal Capacity Analysis

The strong item proven in discern 2. Three is a metallic piece that's being cooled in air after warm forming. Thermal energy is leaving the item from all factors of the surface, and that is proven for simplicity with the aid of an unmarried arrow. The first law of thermodynamics carried out to this trouble is

Heat out of item throughout time dt = Decrease of inner electricity of item at some stage in time dt

Now, if Biot range is small and temperature of the object may be taken into consideration to be uniform,
Then the above equation may be written as

$$hAs[T(t) - T\infty]dT = -\rho CVdT \qquad (2.8)$$

$$or, \quad \frac{dT}{T - T_\infty} = -\frac{hA_s}{\rho cV}dt \qquad (2.9)$$

Integrating and making use of the initial circumstance T (0) = Ti,

$$ln\frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho cV}t \qquad (2.10)$$

Taking the exponents of each sides and rearranging,

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \qquad (2.11)$$

$$Where, \; b = -\frac{hA_s}{\rho cV} \qquad (2.12)$$

Rate of convection warmness transfer at any given time t: $Q(t) = hA_s[T(t) - T\infty]$

Total quantity of warmth transfers among the frame and the encompassing from t=0 to t:

$$Q = mc[T(t) - Ti]$$

Maximum heat switch (limit reached when frame temperature equals that of the encompassing):

$$Q = mc\,[T_\infty - Ti]$$

Before a constant nation circumstance is reached, sure quantity of time is elapsed after the warmth switch manner is initiated to permit the temporary situations to vanish. For example, even as determining the fee of heat waft through wall, we do no longer remember the length at some point of which the furnace starts up and the temperature of the indoors, as well as the ones of the partitions, regularly growth. We commonly expect that this period of transition has surpassed and that constant-state condition has been hooked up.

6.5 Analytical Methods and Numerical Methods:
In general, we employ either an analytical method or numerical method to solve steady or transient conduction equation valid for various dimensions (1D/2D). Numerical technique generally used is finite difference, finite element, relaxation method etc. The most of the practical two dimensional heat problems involving irregular geometries is

solved by numerical techniques. The main advantage of numerical methods is it can be applied to any two-dimensional shape irrespective of its complexity or boundary condition. The heat conduction problems depending upon the various parameters can be obtained through analytical solution. An analytical method uses Laplace equation for solving the heat conduction problems. Heat balance integral method, Hermite-type approximation method, polynomial approximation method, Wiener–Hopf Technique are few examples of analytical methods.

6.5.1 Transient Conduction Analysis in a Square Bar with Specific Heat Flux:

Problem:

We recollect the warmth conduction in a Square bar of thickness 2R, to begin with at a uniform temperature T_0, having heat flux at one aspect and replacing warmth at some other facet. A steady warmness switch coefficient (h) is believed on the other side and the ambient temperature (T_∞) are assumed to be consistent.

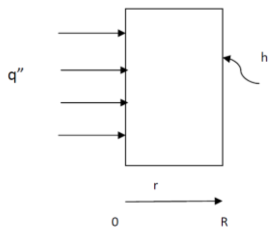


Fig: 9 - Schematic of Square bar with boundary heat flux

Assuming square bar with *a*, the generalized temporary heat conduction legitimates for Square bar, strong cylinder and sphere can be expressed as:

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r^m}\frac{\partial}{\partial r}\left(r^m \frac{\partial T}{\partial r}\right) \quad (4.1)$$

Where, m = 0 for Square bar, 1 and 2 for solid cylinder and sphere, respectively. Here we have considered square bar geometry. Putting m=0, equation (4.1) reduces to

$$\frac{\partial T}{\partial r} = \alpha \frac{\partial^2 T}{\partial r^2} \quad (4.2)$$

Subjected to boundary conditions

$$\frac{\partial T}{\partial r} = -q \quad at \quad r = 0 \quad (4.3)$$

$$k\frac{\partial T}{\partial r} = -h(T - T_\infty) \quad at \quad r = R \quad (4.4)$$

Initial conditions: $T = T_0$ at $t = 0$ (4.5)

Dimensionless parameters are defined as follows

$$\tau = \frac{\alpha t}{R^2},\ \theta = \frac{T-T_\infty}{T_0-T_\infty},\ B = \frac{hR}{k},\ Q = -\frac{q''\delta}{k(T-T_\infty)},\quad x = \frac{r}{R} \quad (4.6)$$

Using equations (4.6) the equation (4.2-4.5) can be written as

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial x^2} \quad (4.7)$$

$$\frac{\partial \theta}{\partial x} = -Q \quad (4.8)$$

Boundary conditions

$$Q = \frac{q''R}{k(T-T_\infty)} \quad at \quad x = R$$

$$\frac{\partial \theta}{\partial x} = -B\theta \quad at\ x = R \quad (4.9)$$

Solution Procedure:

Polynomial approximation technique is one of the only, and in some instances, accurate techniques used to remedy transient conduction problems. The approach involves steps: first, selection of the proper guesses temperature profile, and 2d, to transform a partial differential equation into a variable is average temperature and impartial variable is time. The steps are implemented on dimension less governing equation.

Following guess profile is:

$$\tau = \frac{\alpha t}{R^2},\ \theta_p = a_0(\tau) + a_1(\tau) + a_2(\tau)x^2 \quad (4.10)$$

Differentiating the above equation with respect to x. we get,

$$\frac{\partial \theta}{\partial x} = a_1 + 2a_2 x \quad (4.11)$$

Applying second boundary condition we have

$a_1 + 2a_2 = -B\theta$ (4.12)

Similarly applying first boundary condition at the differentiated equation we have

$$a = -Q \tag{4.13}$$

Thus equation (4.12) may be written as

$$a_2 = \frac{Q - B\theta}{2} \tag{4.14}$$

Using equation (4.8) and (4.9) we get vale of a_0 as:

$$a_0 = -\left(\frac{a_1 + 2a_2}{B}\right) - a_1 - a_2 \tag{4.15}$$

Substituting the values of a_1 and a_2, we get

$$a_0 = \theta + Q - \left(\frac{Q - B\theta}{2}\right) \tag{4.16}$$

Average temperature for long square bar, long solid cylinder and sphere can be

$$\bar{\theta} = \frac{\int_v \theta dV}{\int dV} = \frac{\int_0^1 \theta x^m dx}{\int_0^1 x^m dx} = (m+1)\int_0^1 x^m \theta dx$$

m = 0 for Square bar, 1 and 2 for solid cylinder and sphere, respectively. Here we are using square bar problem.

Hence m=0. Average temperature equation used in this problem is

$$\bar{\theta} = \int_0^1 \theta dx \tag{4.17}$$

Substituting the value of θ and integrating we have

$$\bar{\theta} = \frac{Q}{6} + \left(\frac{1+B}{3}\right)\theta \tag{4.18}$$

Now, integrating non-dimensional governing equation we have

$$\frac{\partial \bar{\theta}}{\partial r} = -B\theta + Q \tag{4.19}$$

Substituting the value of θ we have

$$\frac{\partial}{\partial \tau}\left(\frac{Q}{6} + \frac{1+B}{3}\theta\right) = -B\theta + Q \tag{4.20}$$

We may write the above equation as

$$\frac{\partial \theta}{\partial \tau} + U\theta - V = 0 \tag{4.21}$$

Integrating equation (4.21) we get an expression of dimensionless temperature as

$$\theta = \frac{e^{-U\tau} + V}{U} \tag{4.22}$$

$$\text{Where, } V = \frac{Q}{1 + \frac{B}{3}}, \quad U = \frac{B}{1 + \frac{B}{3}}$$

Based on the analysis a closed form expression involving temperature, heat source parameter, Biot number and time is obtained for a Square bar. Similarly, we can analyze the transient thermal conduction problems for solid cylinder, hollow cylinder and plate.

6.6 Transient Heat Analysis of Conduction

Problems by written as:

using ANSYS Software Tool

6.6.1 ANSYS Software Introduction:

ANSYS is an American Computer-aided engineering software program. ANSYS publishes engineering evaluation software at some stage in a range of disciplines including finite detail evaluation, structural evaluation, computational fluid dynamics, explicit and implicit techniques and heat transfer. Analysis of Temperature distribution in Square bar, Solid cylinder, Hollow cylinder and Plate.

6.6.2 ANSYS - Transient Thermal Analysis - Problem Solving Steps:

1. Create Analysis System
2. Define Engineering Data
3. Attach Geometry
4. Define Part Behavior
5. Apply Mesh Controls/Preview Mesh
7. Establish Analysis Settings
8. Define Initial Conditions
9. Apply Loads / Supports / Heat Flux
10. Solve

VII RESULTS

7.1 Heat Treatment Results of HDS Steels:

7.1.1 Hardening:

HDS Steel (H13) material is kept for 3 hours in the low pressure vacuum hardening furnace & the results graph is Fig.10

7.1.2 1st Tempering:

HDS Steel (H13) material is kept for 3 hours in the low pressure vacuum tempering furnace & the results graph is Fig.11

7.1.3 2nd Tempering:

HDS Steel (H13) material is kept for 3 hours in the low pressure vacuum tempering furnace & the results graph is Fig.12

7.1.4 3rd Tempering:

HDS Steel (H13) material is kept for 3 hours in the low pressure vacuum tempering furnace & the results graph is Fig.13

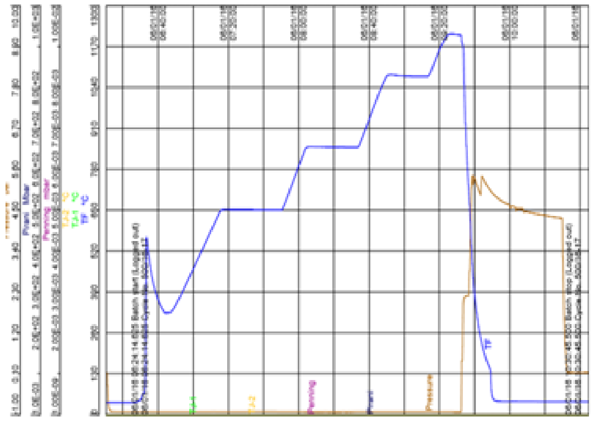


Fig: 10 – Hardening

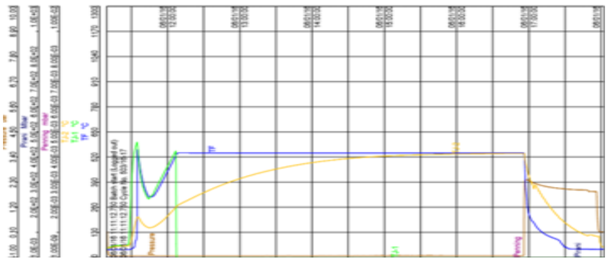


Fig: 11 - 1st Tempering

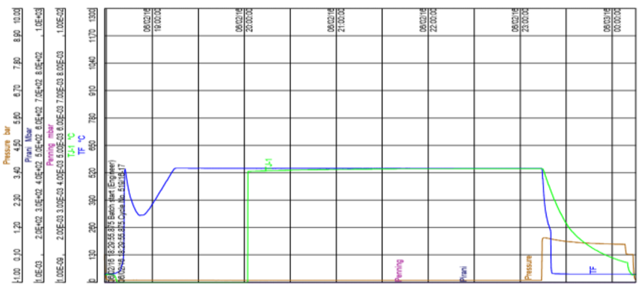


Fig: 12 - 2nd Tempering

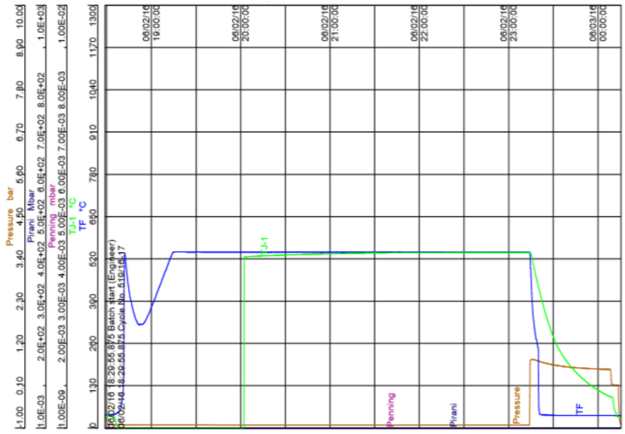


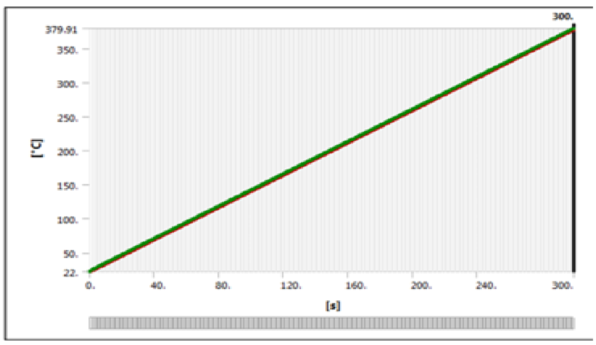
Fig: 13 - 3rd Tempering

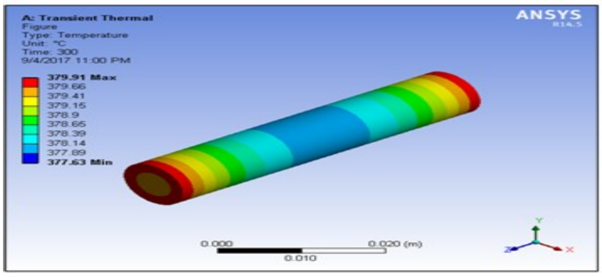
7.2 Heat Transfer Results of Transient Conduction Analysis:

7.2.1 Heat Distribution in Square Bar:

Fig: 14 & 15 shows the version of temperature with time for various warmness supply parameters for a rectangular bar. We take a square bar and then applied a steady warmth flux of 9200W/m^2 on all the faces for 30 seconds with an initial temperature of 22 ^{0}C.

- Fig.14 shows that the temperature on this square bar increased from 22 ^{0}C to 380.23 ^{0}C.
- Fig.15 Shows that the temperature increased from the center (377.57 ^{0}C) to the end (380.23 ^{0}C) of the square bar.





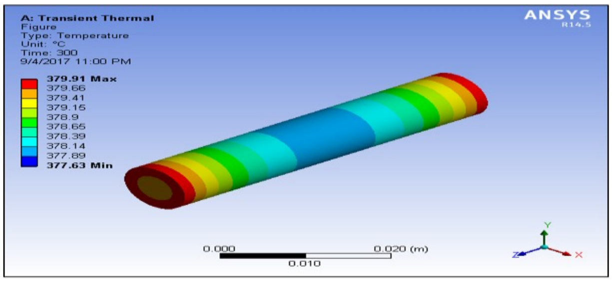


Fig: 14 & 15 - Temperature VS Time and Temperature Distribution - Square Bar

- If we compare the heat transfer formula based result with the Ansys result, a small change in the final temperature.
- Similarly, analyzed for the remaining profiles in Ansys and the results are shown in below.

7.2.2 Heat Distribution in Solid Cylinder:

Fig 16 & 17 shows the variant of temperature with time for various warmness source parameters for a stable cylinder. We take a solid cylinder and then applied a consistent warmth flux of 9200W/m^2 on all the faces for 30 seconds with an initial temperature of 22 ^{0}C.

- Fig.16 shows that the temperature on this square bar increased from 22 ^{0}C to 379.91 ^{0}C.
- Fig.17 shows that the temperature increased from the center (377.63 ^{0}C) to the end (379.91 ^{0}C) of the solid cylinder.

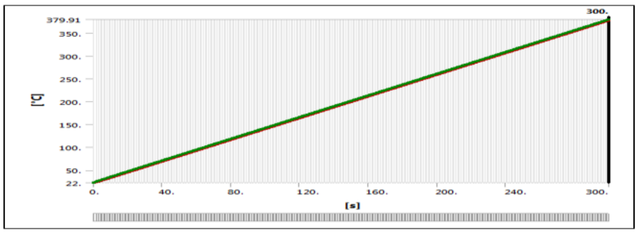
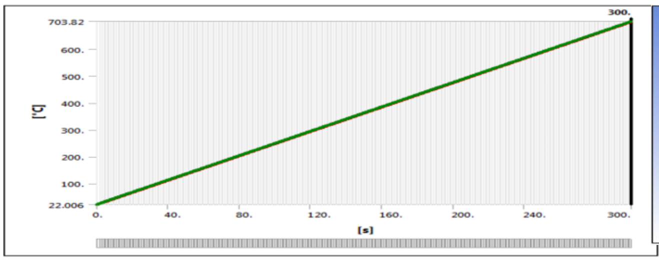


Fig: 16 & 17 - Temperature VS Time and Temperature Distribution – Solid Cylinder

7.2.3 Heat Distribution in Hollow Cylinder:

Fig 18 & 19 shows the variation of temperature with time for diverse heat source parameters for a hollow cylinder. We take a hollow cylinder and then implemented a steady warmness flux of 9200W/m^2 on all the faces for 30 seconds with a preliminary temperature of 22 ^{0}C.

- Fig.18 shows that the temperature on this square bar increased from 22 ^{0}C to 703.82 ^{0}C.
- Fig.19 shows that the temperature increased from the center (701.8 ^{0}C) to the end (703.82 ^{0}C) of the hollow cylinder.



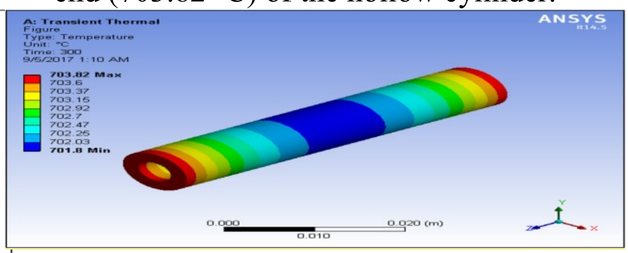
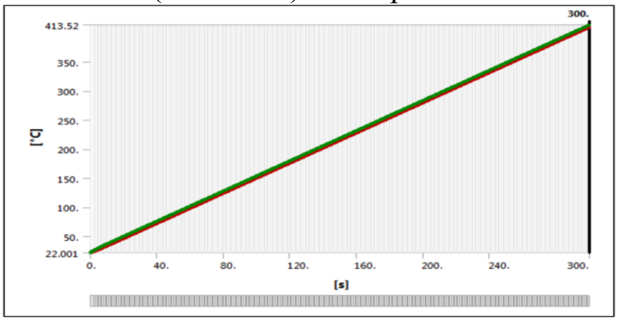


Fig: 18 & 19 - Temperature VS Time and Temperature Distribution – Hollow Cylinder

7.2.4 Heat Distribution in Plate:

Fig 20 & 21 shows the version of temperature with time for diverse heat supply parameters for a plate. We take a rectangular plate, after which applied a steady warmth flux of 9200W/m^2 on all the faces for 30 seconds with an initial temperature of 22 ^{0}C.

- Fig.20 shows that the temperature on this square bar increased from 22 ^{0}C to 413.52 ^{0}C.
- Fig.21 shows that the temperature increased from the center (409.53 ^{0}C) to the end (413.52 ^{0}C) of the plate.



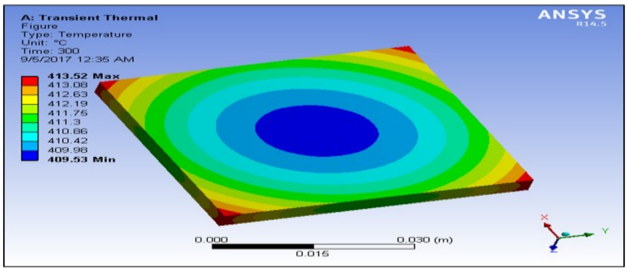


Fig: 20 & 21 - Temperature VS Time and Temperature Distribution – Plate

7.3 Heat Transfer Result of Transient Conduction Problems:

7.3.1 Tabulation:

Comparison of solutions of average temperature obtained from different heat conduction problems:

S. No.	Profile Type	Average Temperature
1	Square Bar	$\theta = \frac{e^{-U\tau}+V}{U}$, $V = \frac{Q}{1+\frac{B}{8}}$ & $U = \frac{B}{1+\frac{B}{8}}$
2	Solid Cylinder	$\theta = \frac{\exp(-P\tau)}{1+\frac{B}{4}}$, $P = \frac{8B}{4+B}$
3	Hollow Cylinder	$\theta = \frac{e^{-U\tau}+V}{U}$, $U = \frac{B}{(4+B)/8}$ & $V = \frac{Q}{(4+B)/8}$
4	Plate	$\theta = \frac{e^{-U\tau}+V}{U}$, $V = \frac{Q}{1+\frac{B}{8}}$ & $U = \frac{B}{1+\frac{B}{8}}$

7.3.2 Tabulation:

Comparison of modified Biot number against various temperature profiles for a bar & plate:

S. No.	Profile Type	Value of P
1	$\theta_p = a_0(\tau) + a_1(\tau) + a_2(\tau)x^2$	$P = \frac{3B}{3+B}$
2	$\theta_p = a_0 + a_1(x^2 - x) + a_2(x^3 -$	$P = \frac{13B}{13+12B}$
3	$\theta_p = a_0 + a_1(x^4 - x^2) + a_2(x^3 -$	$P = \frac{30B}{30+17B}$
4	$\theta_p = a_0 + a_1(x^4 - x^2) + a_2(x^2 -$	$P = \frac{30B}{30+13B}$
5	$\theta_p = a_0 + a_1(x^4 - x) + a_2(x^5 -$	$P = \frac{24B}{24+13B}$
6	$\theta_p = a_0 + a_1(x^4 - x^3) + a_2(x^4 -$	$P = \frac{20B}{20+21B}$

7.3.3 Tabulation:

Comparison of modified Biot number against various temperature profiles for cylinders:

S. No.	Profile Type	Value of P
1	$\theta_p = a_0(\tau) + a_1(\tau) + a_2(\tau)x^2$	$P = \frac{8B}{4+B}$

2	$\theta_p = a_0 + a_1(x^2 - x) + a_2 x^2$	$P = \frac{4B}{2+B}$
3	$\theta_p = a_0 + a_1(x^3 - x) + a_2 x^3$	$P = \frac{30B}{15+3B}$
4	$\theta_p = a_0 + a_1(x^4 - x) + a_2(x^3 - x$	$P = \frac{10B}{5+4B}$

VIII CONCLUSION

An improved lumped parameter model is applied to the transient heat conduction in a long plate and long solid cylinder. Polynomial approximation method is used to predict the transient distribution temperature of the plate and hollow cylinder geometry. Four different cases namely, boundary heat flux for both plate and hollow cylinder and, heat generation in both plate and hollow cylinder has been analyzed. Additionally, different temperature profiles have been used to obtain solutions for a plate.

A unique number, known as modified Biot number is, obtained from the analysis. It is seen that the modified Biot number, which is a function of Biot number, plays important role in the transfer of heat in the solid. Based on the analysis the following conclusions have been obtained.

1. Initially a plate subjected to heat flux on one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis, a closed form solution has been obtained.

2. A long solid cylinder subjected to heat flux on one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis, a solution has been obtained.

3. A plate subjected to heat generation at one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis, a closed form solution has been obtained.

4. A long solid cylinder subjected to heat generation at one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis a closed form solution has been obtained.

IX REFERENCES

[1] Jian Su, "Improved lumped models for asymmetric cooling of a long plate by heat convection", Int. Comm. Heat Mass Transfer, 28, (2001), 973-983
[2] E.J. Correa and R.M. Cotta, "Enhanced lumped-differential formulations of diffusion Problems", Applied Mathematical Modelling 22 (1998) 137-152
[3] Heat Transfer Text Book by Sachdeva.
[4] Ge Su a, Zheng Tan and Jian Su b, "Improved lumped models for transient heat conduction in a plate with temperature-dependent thermal conductivity, Applied Mathematical Modeling, 33 (2009), 274–283
[5] H. Sadat, "A general lumped model for transient heat conduction in one dimensional geometries", Applied Thermal Engineering, 25, (2005) 567–576
[6] F. de Monte, "Transient heat conduction in one-dimensional composite plate. A `natural' analytic approach", International Journal of Heat and Mass Transfer, 43, (2000), 3607-3619
[7] James R. Kingsley-Rowe, Gary D. Lock and J. Michael Owen, "Transient heat transfer measurements using thermos chromic liquid crystal: lateral-conduction error", International Journal of Heat and Fluid Flow, 26, (2005), 256–263
[8] O¨. Ercan Attar, "An approximate method for transient behavior of finned-hollow cylinder cross flow heat exchangers", International Journal of Refrigeration, 27, (2004), 529–539