COMPUTATIONAL STUDY OF FLOW OF BLOOD THROUGH VISCOELASTIC SYSTEM IN A CAPILLARY

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Abstract

The viscoelastic systems are computationally studied in a pressure driven flow with sudden contraction and expansion using finite difference method. The effects of viscoelasticity in drop and bulk fluids are investigated including high Weissenberg and Reynolds number the finely extensive model is used to account for the fluid viscoelasticity. Extensive computations are performed to examine drop dynamic for a wide range of parameters. It is found that the viscoelasticity interact with drop interface in anonmonotonic and complicated ways and a two phase viscoelastic systems exhibits very rich dynamic especially in the expansion region at high Re, the drop undergoes large deformation for a highly viscous drop a re-entrant cavity develops in the contraction region at the trailing edge which in certain cases groups and eventually caused encapsulation of ambient fluid.

Keywords - *ambient fluid, extensive computations, bulk fluid, finite difference, extensive model.*

Introduction

Confined multiphase fluid dynamics is a fundamental importance in wide range of engineering application and natural processes such as the droplet based micro fluids [3, 4] enhanced oil recovery [5] blood cells in micro circulation [6-8], Polymer blend and polymer processing [10] viscoelectric effect plays a significant role in all these application often in the presence of confinement viscoelastic liquid exhibit range of exotic behaviours that can be utilized to perform useful functions especially in microfluidic applications for instance micro fluidic memory and control device [11] micro fluidic rectifier [12], nonlinear viscoelastic flow resistor, [11] synthesis of non-spherical partial [13] and enhance mixing in micro channels [14] rely on viscoelasticity of working fluid with the rapidly growing popularity of droplet based micro fluidic devices there has been significant interest recently in viscoelastic two phase system in confined geometries.

Unlike viscous system and understanding of viscosity is severally limited which show great importance in flow modelling and simulations. In uniform extensional flow reduced deformation is predicted for a viscoelectric drop in a viscous medium and enhance deformation, this is consistent with the neurastic idea with a viscoelestacity in the drop phase which opposes deformation also there is contrary result about the effect of fluid elasticity on drop deformation in a converging conical channel.

The confined multiphase flows have recently received a special attention due to growing interest in micro fluidic technologies. Some researcher's works on this field [20-21] they demonstrated the viscoelasticity of the suspending flow significantly influences the droplet dynamics in straight and constricted capillary tube. The recent study [22] studied deformation and breakup of viscoelastic two phase system in capillaries of different cross sections for cylindrical and rectangular channels an increase in elasticity of drop phase inhibits deformation while in suspending fluid it has non monotonic effect a decreased followed by increased. They also investigated that drop dynamic in periodically constricted capillary tubes and found that drop deformation is enhanced by viscoelasticity irrespective of which phase is elastic and concluded that viscoelasticity in the interior phase pulls the trailing end inward while in exterior fluid, pull it out.

In the present study extensive direct numerical simulations are carried out to examine the effect of visco elasticity on drop dynamics in various two phase system. In pressure driven tube with sudden contraction or expansion. The inertial effects are fully taken into account by solving Navier stokes and viscoelastic model equation in the entire computational domain using the front tracking method.

Formulation and Numerical Method :

A single set of governing equation can be written for the entire computational domain provided that they jumps in the material properties including density, viscosity and relaxation time are taken into account and the effects of interfacial surface tension are treated appropriately.

The continuity and the momentum equation can be written as follows.

$\nabla.u = 0$------------- (1)

$\frac{d\rho u}{dt} + \nabla (\rho uu) = -\nabla p+\nabla.\mu s(\nabla u+\nabla u^T)+\nabla.E+\int_A \sigma k n\delta(x-xf)dA$-------- (2)

Where μs μs - solvent viscosity

ρ - density of fluid

p - Pressure

u - velocity vectior

τ - visco elastic tensor

The FENE-CR model is adopted as the consecutive equation for the viscoelastic extra stress then this model can be written as.

$\frac{dA}{dt} + \nabla. (uA) -(\nabla u)^T.A-A.\nabla u=\frac{f_A}{\lambda}(A-I)$

$F_A = \frac{L^2}{L^2-track\ (A)}$-------- (3)

Where,

A - Confirmation tensor

λ - relaxation time

L - equilibrium length

F_A - Stretch function

I - identify tensor

The extra stress tensor τis related to the confirmation tensor as

$\tau=\frac{F_A \mu_p}{\lambda}(A-I)$ ----------------- (4)

μp- polymeric viscosity.

It is also assumed that the material properties remain constant following a fluid partial.

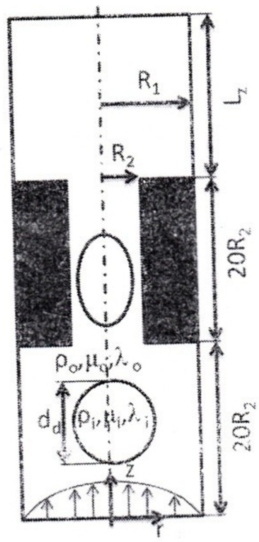
$\frac{D_\rho}{D_t} = 0, \quad \frac{D_{\mu s}}{D_t} = 0, \frac{D_{\mu p}}{D_t} = 0, \quad \frac{D\lambda}{D_t} = 0$ ------------- (5)

The flow equation [1,2] are solved fully coupled with the viscoelastic model equation [3]. The flow and viscoelastic model equation are solved on a staggered euterian grid using a projected method.

A separated Lagrangian grid is used to explicitly track the fluid. Fluid interface. The Lagrangian grid consists of linked marker point moving the local flow velocity interpolated from stationary euterian grid. The piece of Lagrangian grid between two marker point is called a front element. The surface tension nis computed on the Lagrangian grid points near the interface to be added to the momentum equation.

Problem Statement:

The physical problem and computational domain are seen in dia 1 the flow is assumed to be axisymmetric. So only one half is used as computational domain the capillary tube contains 4:1:4 contraction/expansion section the radii of the main channel and the contraction are R_1& R_2. The total length of bube is varied depending on the Raynolds number (Re). At higher Re due to oscillation of the drop in the expansion region, longer tube is required to reach steady state the flow is initiated instantaneously by imposing a fully developed steady flow at the inlet and keeping the pressure constant at the outlet. Symmetry and noslipboundry condition are applied at the centreline and at the wall of the tube respectively, the elastic stress tensor at the inlet is specified based on analytical solution assuming a fully developed pipe flow the Neumann boundry conditions are used for all viscoelastic stress component at the other boundaries. The governing equations are solved in their dimensional from but the results are expressed in terms of relevant non-dimensional quantities.



The governing no-dimensional number are defined as–

Ca=$\frac{\mu o v}{\sigma}$, Re=$\frac{\rho_0 V R_2}{\mu_0}$, Wi=$\frac{\lambda V}{R_2}$, $\theta = \frac{\mu i}{\mu_0}$,

α=$\frac{\rho i}{\rho o}$, K=$\frac{\lambda_i}{T_0}$, B=$\frac{\mu s}{\mu_\rho + \mu_s}$, K= $\frac{dt}{2R_2}$,

Where, Ca = Capillary
Re = Reynolds number
Wi = Weissenberg number
O = Viscosity
α = density
K = relaxation time
B = Solvent Viscosity

& the drop deformation is defined as deformation = $\frac{d_a - d_r}{d_a + d_r}$

Where, da- maximum droplet dimension in the axial direction.

dr- maximum droplet dimension in the radial direction.

Result &Discussion:

Extensive simulations are carried out to study the dynamics of viscoelastic two phase system in a pressure driven axisymectric contraction / expansion capillary tube a non-uniform Cartesian grid (dia-1) is employed in the computations. The grid is stretched near the contraction and expansion region, where the largest stress gradient occur. A comprehensive grid convergence study is performed to determine the minimum grid size required to reduced the spatial discretization error below a threshold value.

The simulations are first performed to examine the effects of various flow parameters on the drop dynamics in the NV, VN and VV system at flow Reynold number i.e. Rc=2 then the further simulations are performed to demonstrate the combined effect of inertia and viscoelasticity for the Reynolds and Weissenberg numbers up to Re=100 & Wi=100.

(A) Computations at the low Reynolds number (Re=2)

(1) Effects of confinement (k)

First the effect of confinement on the drop dynamics are examined in the constriction region the confinement is characterized by the relative droplet size K and it changes between 0.5 ≤ k ≤ 1.73 while keeping all other parameters fixed at their values in the base case in this it is seen that the curvature at the fore is higher than that at the aft, due to pressure gradient in the axial direction. Large viscoelastic stresses are generated at the side of the drop due to high shear stress exerted by continuous fluid and then these stresses are convected by internal circulation to create a stress concentration near the rear stagnation point to the plane case study the droplet takes a bullet like shape at the low confinement and become more elongated as k increases.

(2) Effect of Weissenberg number (Wi)

We investigate the viscoelastic effect on drop dynamics at Re=2 for low Re viscoelastic effects are especially pronounced in the constriction region. It is said that the drop shape is strongly influenced by viscoelastic normal stress difference. The drop shapes together with the counters of constant normal stress differences for NV & VN cases for various combinations of Ca&Wi numbers. Since the steady droplet shape is not attained by the end of constriction for high Ca and Wi number cases. The results are shown when the droplets are approximately at the middle of constriction it is also seen that the increase in maximum normal stress difference gets smaller when Wi exceeds a threshold value and the decreasing surface tension enhances drop deformation and reduces strain rate inside the droplet, thus for higher Ca the viscoelastic stresses inside the drop grow more slowly. The drop deformation is affected farless by the viscoelasticity than the capillary forces. The viscoelastic stresses inside the drop at the rear stagnation point pull the interface inward resulting in slight indentation at the trailing edge as ca increases the droplet becomes more elongated and its leading edge bulges the bulge formation is caused by surface tension that increases the pressure inside the curvature at the tip of the interface and in turn pulls the tip towards the main body of droplet.

(3) Effect of Total Viscosity ratio -

In the current study we examine the effects of drop to matrix viscosity ratio θ. It is seen that the middle of the constriction show great sensitivity of drop deformation to θ as the θ increase. The viscous normal stresses grow and eventually dominate over the combination of viscoelastic normal stresses and surface tension leading to development of re-entrant covity in the straight capillary tube the viscoelasticity in drop matrix fluid generally enhances drop deformation. The weissenberg number is increased for better demonstrate the stress distribution in the vicinity of droplet as just before entering the constriction a viscoelastic stress concentration occurs at the back of the droplet. Pushing the trailing edge to cause an indentation and subsequently a re-entrant cavity formation in the NV case on the other hand in the VN case the viselastic stress concentration occurs at the shoulder of the droplet that act against the viscous stresses to restore the indentation of the trailing edge.

It is evident that viscoelastic effects are important for the development of a puckered end a re-entrant cavity. It is also observed that the viscoelasticity has a non-monotonic effect on the drop deformation i.e. the size of the re-entrant cavity and resulting inner droplet first decreases and then increases with Wi. The behaviour is mainly attributed to the fact that the polymer stresses take a finite time to react to the flow. The effects of viscosity ratio on drop deformation for both VN & NV cases. The drop deformation increases with the viscosity ratio in both system with similar trend but a drop undergoes larger deformation in the NV system than that in the VN system in the constriction region a less viscous drop reaches a steady state very quickly while a more viscous drop continuously elongates and exists the constriction without reaching a steady motion. In the capillary number is below the critical value for the breakup and the narrow section is sufficiently large.

(4) Effect of Extensibility (L^2) & Solvent Viscosity

The solvent viscosity ratio (β) ratio essentially, modified and effective. Weissenberg number defined as Wi'=Wi (1-β). The increasing β has a similar effect as decreasing the Weissenberg Number. The drop deformation generally increases as β decreases and the effects of β are moderate which shows the drop deformation is not very sensitive to Wi the larger the L^2 the more the molecule can be extended leading to enhanced visco increases with L^2 which in turns to increases the deformation of the drop.

(B) Computations at High Reynolds Number:

The Reynold number and fluid elasticity ratio are fixed at Re=100 & K=0.2 for both cases while the inner and outer Weissenberg number are selected differentially to demonstrate the effects of viselasticity at the trailing tip, the surface tension and viscoelastic stresses inside the droplet oct in the same direction to increase the radius of tip and thus to enhance the bulge formation. The viscous and the viscoelastic stresses in the ambient fluid are directed away from the bulk of the droplet promoting a breakup while viscoelastic stresses stress in the drop fluid and the surface tension act to pull the bulge back towards the bulk of the droplet and thus suppressing the breakup strong interaction between the inertia surface tension and viscoelastic stresses result in very rich and highly complicated dynamics both in constriction and expansion regions, the droplet undergoes large deformation in the narrow channel followed by shape oscillations int he expansion region. The oscillations sustain for a long time in the downstream of the expansion.

Conclusion :

The effects of viscoelasticity on drop dynamic and deformation in pressure driven capillary tube with sudden contraction/ expansion are studied computationally wing front tracking Method. A Newtonian drop in a viselastic medium (NV), a viscoelastic drop in a Newtonian medium (VN) and a viscoelastic medium (VV) case are considered. Extensive simulations are perform to examine the effect of revalentdiamensions parameters including the viscosity ratio (θ), the relative droplet size (k) the reynold number (Re) and the capillary number (Ca) as well as the fluid elasticity Charachterised by Weissen berg number (Wi) solvent viscosity ratio (β) and the extensilble parameter (L^2) in the VV case the effects of inner and outer Weissenberg number as well as their ratio (k) are also examined.

At high Re, the droplet exhibits very rich dynamic with highly complicated deformation and stress distribution pattern. In particular the droplet undergoes large deformation inthe constriction followed by damped surface oscillation in the expansion region the magnitude of surface oscillations first increased/ decreased and then increased/ decreased with Wi for the VN/NV case the large deformation may even lead to a drop breakup.

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