

## A MATHEMATICAL PERSPECTIVE ON DYNAMICAL CHARACTERISTICS OF BLOOD FLOW IN HUMAN ARTERIES

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### ABSTRACT

*The effects of an overlapping stenosis on blood flow characteristics in an artery have been studied. Blood has been represented by a couple stress fluid is observed that the resistance to the flow, pressure drop and wall shear stress increases with height and length of the stenosis. And it is noticed that the resistance to the flow and pressure drop decreases with couple stress fluid parameters. The effect of different physical characteristics on liquid impedance to flowing, velocity, and surface shearing stress are studied. Herschel–Bulkley equation has been taken to represent the non-Newtonian character of blood. The problem is investigated by a combined use of analytical and numerical techniques. The shape, length and height of the stenosis in an artery under various flow conditions play a pivotal role. Research has been carried out by many researchers on the nature of geometry of the stenosis in the artery, nature of the blood and its flow. Finite difference scheme has been used to solve the unsteady nonlinear Navier–Stokes equations in cylindrical coordinate's system governing flow assuming axial symmetry under laminar flow condition so that the problem effectively becomes two-dimensional. The relations for surface shearing stress, pressure drop, flow resistance, and velocity are calculated analytically by applying a mild stenosis approximation.*

**Keywords:** blood flow, flow resistance, Herschel–Bulkley, flowing, velocity, two-dimensional.

### INTRODUCTION

Several investigations have shown that the rate of Kolmogorov spectrum formation is reduced by helicity, where a non-zero

helicity partially inhibits the energy flux from larger to smaller scales. Furthermore, the stability induced by spiral flow and the decrease in laterally directed forces may reduce turbulence in the tapering branching arterial tree and at stenoses. Many properties of systems presenting helicity are related to the reduction of non-linear processes responsible for the transfer and redistribution of energy through various scales. The proposed function, computed along a cluster of particle trajectories, has been tested for the quantitative determination of the spiral blood flow in a three-dimensional, s-shaped femoral artery numerical model in which three degrees of stenosis were simulated in a site prone to atherosclerotic development. Our results confirm the efficacy of the Lagrangian analysis as a tool for vascular blood dynamics investigation. Several biomedical engineers also concentrate on several other elementary engineering fields, like controls and systems difficulties via development of breakthrough rehabilitation, medical imaging and disease detection equipment, among others. Therefore, the essence of biomedical engineering has many



applications due to the necessity of understanding various engineering concepts and physiology and applying insights from both areas to study. Main objective of biomedical engineering is to combine these subjects in order to understand these mechanisms or to design and manufacture devices for use in a biological or medical equipment.

Similarly, if one has to create a novel implanted a device for cardiovascular system, then there is a need of understanding and evaluating not just mechanical flow principles, but also material qualities, electrical components, and the physiological implications the device can have on the circulatory system. Therefore, this type of difficulty gets close to the core of a biomedical engineer's duties, which are to create equipment to rectify physiological state which to quantify the effect of such equipment in physiologically relevant contexts. Many biomedical engineers are only concerned with engineering design, while others are more concerned with physiological applications. The peculiarity of Beltrami flows is that their flow fields correspond to a state of maximum kinetic helicity, for a given total kinetic energy.

#### LITERATURE REVIEW

**Shabab Akbar (2024)** the study of fluid dynamics is crucial to understanding fluid flow in the human body, and cardiovascular physics places a lot of concentration on blood flow modeling. Nevertheless, the models that have been created thus far with three-dimensional analysis are extremely complex. This research work offers study of blood pressure, flow and other flow-related parameters. Modeling artery was an extensible circular pipe with oscillating

blood flow. Understanding factors that could lead to high blood pressure as one of the reasons for studying blood flow. The cardiovascular system equation is a straight forward differential equation that was developed under certain assumptions using Navier-Stokes equations. Generic study of normal blood flow was then created by applying some assumptions to the equation for the cardiovascular system.

**Annord Mwapinga (2024)** Computational modeling of arterial blood flow, heat and mass transfer during MRI scanning is studied. The flow is assumed to be unsteady, in-compressible, and asymmetric. Mathematical formulation considers the presence of stenosis, joule heating viscous dissipation and chemical reaction. The explicit finite difference scheme is used to numerically solve the model equations. The MATLAB software was used to plot the graphical results. The study reveals that, during MRI scanning, both radial and axial velocities diminish with increase in the strength of magnetic fields. Besides, the study found that, Eckert number and Hartman number enhance the blood's temperature and the same, diminishes with increase in Prandtl and Reynolds numbers.

**Z S Othman (2021)** an interface between biology and mathematics has initiated and fostered new mathematical areas, where the ideas from biology and mathematics are synergistically applied. Study of fluid dynamics plays a significant role in fluid flow inside the human body, and modeling of blood flow is an important field in circulatory physics. However, models have been developed are very complex with three-dimensional analysis. This project presents a novel and simple mathematical model of blood flow. Assuming blood is a

Newtonian fluid which is governed by the Navier-Stokes equations and continuity equation and with making use of the Navier-Stokes equation, a simple differential equation called as the circulatory system equation is derived. Then by applying the logical assumptions on this model, the general mathematical model of the normal blood flow rate is developed.

**C. Umadevi (2021)** A literature survey related to nanoparticles reveals that nanofluids are getting more popularity in hematological treatment. Development in this direction motivated us to write a theoretical study on the flow of blood mixed with suspended copper nanoparticles with water as a base fluid in an inclined overlapping stenosed artery under the influence of a magnetic field. The geometry of an overlapping stenosed arterial section is being written mathematically by an appropriate geometric expression. The rheology of the blood is formulated mathematically by coupled partial differential equations. The coupled partial differential equations governing the problem are simplified and solved analytically under mild stenosis assumptions. The expressions for velocity, temperature, pressure droplet, flow resistance, and wall shear stress distribution are obtained in the present analysis.

**Blessy Thomas (2016)** The blood flow in human arterial system can be considered as a fluid dynamics problem. Simulation of blood flow in the arterial network system will provide a better understanding of the physiology of human body. Hence, hemodynamics plays an important role in the development and progression of arterial stenosis, leading to the

malfunctioning of cardiovascular system. Simulation studies of blood flow in the diseased condition can diagnose the health problem easily and also have many applications in the areas such as surgical planning and design of medical devices. This study presents a review on the existing scenario of the simulation studies of blood flow, starting with a brief overview of the structure and function of arteries and veins followed by a discussion on pressure wave propagation, blood flow models and Fluid Structure Interaction in the arterial system.

### **Complexity of Blood**

The blood flow has a complex structure in the cardiovascular system. This flow is more complex than the flow encountered in hemodynamic for the following causes:

- Blood flows in various numbers of interconnected arteries of varying diameters and magnitudes.
- Blood flow is pulsatile due to the periodic nature of pumping by the heart.
- Blood is not a homogeneous liquid. It is having plasma with suspensions of RBC, WBC, and platelets, etc. Some of the tubes have diameters less than the size of RBC so that the cell is deformed during the movement.
- Blood is not a Newtonian liquid and its stress-strain relation cannot be articulated by a simple method.

### **Flow Rate of Blood**

Movement through a blood tube is determined by two features: the force that presses on the blood through the tube, and the flexibility of the tube to the blood flow. In general, the rate of fluid movement is calculated in milliliters or liters per minute. The flowing movement in creature

beings is about 5000 ml/min at rest but may be approximately 5-6 times increases during heavy physical exercise when the bodies require more oxygen to fuel that workout. The blood amount pumped out by the heart is considered the cardiac output. It is important that the flow in the creature's body is influenced by the earth's gravity.

When a creature is standing, gravity assists pull the blood sliding to the inferior extremities. Without the earth's gravity, blood remains close to the heart. The gravity makes it more complicated for the blood to move upward to reverse to the heart and lungs for O<sub>2</sub>. Creature bodies have grown to deal with the ever-present downward strength of earth gravity; leg muscles function as second pumps to assist in blood flow return to the heart, also known as cardiac input.

## Types of Blood Vessel

### Artery

Arteries are such blood vessels that lead blood to all the body organs. While most arteries bring oxygenated liquid, there is one pair of exceptions, the umbilical and pulmonary blood arteries. Arteries are having more strength than veins. Very big arteries extend when the pressure of liquid rises at some stage in systole and thereafter recoil diastole. The flexible recoil of the cells facilitates to generate a smoother movement of blood in the small arteries and arterioles. Small arteries and arterioles are less flexible than the big artery diameter and have a proportionally broad layer of the smooth cell.

### Vein

Veins are big in diameter and thin layer vessels that transport blood from each tissue in the body to the heart. They are

distensible and enclose a large fraction of blood volume. Blood is used to lose almost it is pressure in our capillaries so it has low pressure within veins and thereafter flows slowly. Veins do not necessitate broad layers and have a better lumen comparison to an artery, to decrease the resistance of blood movement.

### Capillary

Capillaries are the modest controlled vessels. Capillaries are enormously thin layered vessels that allow the exchange of various gases, nutrients and many other small molecules amid the blood movement and cells. They are a collection of a unique cell layer of endothelium and approximately 8 mm in modulus.

### Sinusoid

A sinusoid is one type of capillary alike to fenestrated endothelium. Sinusoids are really classified as free pore capillary as opposed to regular and fenestrated kind. Fenestrated capillaries contain diaphragms that surround the pores while open-pore capillaries require a diaphragm and have an unlock pore. The free pores of endothelial units greatly raise their permeability.

### Mathematical model

A mathematical model is an abstract description of a concrete system using mathematical concepts and language. The process of developing a mathematical model is termed mathematical modeling. Mathematical models are used in applied mathematics and in the natural sciences (such as physics, biology, earth science, chemistry) and engineering disciplines (such as computer science, electrical engineering), as well as in non-physical systems such as the social sciences (such as economics, psychology, sociology,



political science). It can also be taught as a subject in its own right.

The use of mathematical models to solve problems in business or military operations is a large part of the field of operations research. Mathematical models are also used in music, linguistics, and philosophy (for example, intensively in analytic philosophy). A model may help to explain a system and to study the effects of different components, and to make predictions about behavior.

### **Elements of a mathematical model**

Mathematical models can take many forms, including dynamical systems, statistical models, differential equations, or game theoretic models. These and other types of models can overlap, with a given model involving a variety of abstract structures. In general, mathematical models may include logical models. In many cases, the quality of a scientific field depends on how well the mathematical models developed on the theoretical side agree with results of repeatable experiments. Lack of agreement between theoretical mathematical models and experimental measurements often leads to important advances as better theories are developed. In the physical sciences, a traditional mathematical model contains most of the following elements:

1. Governing equations
2. Supplementary sub-models
  - Defining equations
  - Constitutive equations
3. Assumptions and constraints
  - Initial and boundary conditions
  - Classical constraints and kinematic equations

### **Hemodynamics**

Hemodynamics or haemodynamics are the dynamics of blood flow. The circulatory

system is controlled by homeostatic mechanisms of autoregulation, just as hydraulic circuits are controlled by control systems. The hemodynamic response continuously monitors and adjusts to conditions in the body and its environment. Hemodynamics explains the physical laws that govern the flow of blood in the blood vessels.

Blood flow ensures the transportation of nutrients, hormones, metabolic waste products, oxygen, and carbon dioxide throughout the body to maintain cell-level metabolism, the regulation of the pH, osmotic pressure and temperature of the whole body, and the protection from microbial and mechanical harm.

Blood is a non-Newtonian fluid, and is most efficiently studied using rheology rather than hydrodynamics. Because blood vessels are not rigid tubes, classic hydrodynamics and fluids mechanics based on the use of classical viscometers are not capable of explaining haemodynamics. The study of the blood flow is called hemodynamics, and the study of the properties of the blood flow is called hemorheology.

### **Blood Rheology**

Rheology is the study of deformation and flow of a material. We say that a body has been deformed if its shape or size has been altered due to the action of appropriate forces. If the degree of deformation changes continuously with time, the body is considered to be flowing.

This section deals with hemorheology: the science of deformation and flow of blood and its formed elements. This field includes investigations of both macroscopic blood properties using rheometric experiments as well as microscopic properties in vitro and in vivo.

Hemorheology also encompasses the study of the interactions among blood components and between these components and the endothelial cells that line blood vessels.

Advances in the field of hemorheology are based on the evidence that they might be the primary cause of many cardiovascular diseases. In fact, hemorheological aberrations can easily be considered as a result (or an indicator) of insufficient circulatory function. Basically, pathologies with hematological origin like leukemia, hemolytic anemia, thalassemia or pathologies associated with the risk factors of thrombosis and atherosclerosis like myocardial infarction, hypertension, strokes or diabetes are mainly related to disturbances of local homeostasis. Therefore, the mathematical and numerical study constitutive models that can capture the rheological response of blood over a range of flow conditions are ultimately recognized as an important tool for clinical diagnosis and therapeutic planning. To better interpret and analyze the experimental data on blood it is helpful to turn to the literature on the rheology of particle suspensions. For rigid particles, a vast amount of published literature exists. However, the study of suspensions of multiple, interacting and highly deformable particles such as blood, has received less attention and presents a challenge for both theoretical and computational fluid dynamicists.

#### **Non-Newtonian properties of blood**

The mechanical properties of blood should be studied by considering a fluid containing a suspension of particles. A fluid is said to be Newtonian if it satisfies the Newton's law of viscosity (the shear stress is proportional to the rate of shear

and the viscosity is the constant of proportionality). Blood plasma, which consists mostly of water, is a Newtonian fluid. However, the whole blood has complex mechanical properties which become particularly significant when the particles size is much larger, or at least comparable, with the lumen size. In this case, which happens at the microcirculation level (in the small arterioles and capillaries) blood cannot be modelled as a homogeneous fluid and it is essential to consider it as a suspension of blood cells (specially RBCs) in plasma. The presence of the blood cellular elements and their interactions leads to significant changes in the blood rheological properties and reliable measurements need to be performed to derive appropriate microstructural models. Otherwise, depending on the size of the blood vessels and the flow behaviour, it is approximated as a Navier-Stokes fluid or as a non-Newtonian fluid. Here we assume that all macroscopic length and time scales are sufficiently large compared to length and time scales at the level of the individual erythrocyte so that the continuum hypothesis holds. Thus the models presented here are not appropriate in the capillary network.

#### **CONCLUSIONS**

Simulation studies conducted based on the developed mathematical model offer valuable insights into the behavior of blood flow under diverse conditions. The intention of this work was to discuss the result of many parameters on the physiologically significant flow quantities like Slip Condition, Magnetic Effect, Shear Stress, Length of Constriction, etc. The governing important parameters for the blood flow has found so this



hemodynamic study will be helpful and beneficial to medical practitioners, biostatisticians, bio-medical engineering and bio-mathematicians. The main aim of this project is to develop a mathematical model of blood flow and extended the model for two -fluid blood flow through artery. These simulations enable researchers to explore the effects of varying parameters on blood flow dynamics and pressure regulation. Through comparisons between simulated results and experimental data, researchers can ascertain the accuracy and predictive capabilities of mathematical models. An emerging area with promising applications in therapeutic simulation and medical image analysis involves the creation of computerized models depicting human organs.

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