

ROLE OF THE DIFFERENTIAL TRANSFORM METHOD IN MODELLING REAL-WORLD DYNAMICAL SYSTEMS

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ABSTRACT

This study investigates the use of the differential transform method (DTM) for integrating the Rössler system of the fractional order. Preliminary studies of the integer-order Rössler system, with reference to other well-established integration methods, made it possible to assess the quality of the method and to determine optimal parameter values that should be used when integrating a system with different dynamic characteristics. Among different numerical and semi-analytical methods for solving initial and boundary value problems, the differential transform method (DTM) has received notable attention. It has developed and experienced generalizations for implementing other types of mathematical problems such as optimal control, calculus of variations, and integral equations. The first method is based on the differential transform method (DTM), which seeks an analytical solution in the form of a certain functional series. The second method, on the other hand, is based on the physics-informed neural network (PINN), where artificial intelligence in the form of a neural network is used to solve the differential equation. In addition to describing both methods, this study also presents numerical examples along with a comparison of the obtained results. Comparing the two methods, DTM produced marginally more accurate results than PINNs. While PINNs exhibited slightly higher errors, their performance remained commendable.

Keywords: differential transform method (DTM), different numerical and semi-analytical methods, artificial intelligence, well-established integration methods, optimal control.

INTRODUCTION

In these systems, the higher derivative of the unknown cannot, in general, be easily isolated due to nonlinearities. Although these systems can offer greater flexibility in

modeling complex problems, they can be more challenging to solve and require more advanced numerical methods. Furthermore, applying implicit numerical integration methods to such systems will require the solution of nonlinear algebraic systems at every step. This will involve a large computational effort, especially for large-dimension systems. Finding analytical solutions of partial differential equations (PDE) is of vital importance because of their wide range of applications in engineering, medicine and other sciences. Various methods have been devised for this purpose such as the classical methods, which are particularly applicable to linear PDEs, and semi analytical methods such as the Adomian Decomposition Method (ADM), the Homotopy Perturbation Method (HPM), and the Differential Transformation Method (DTM), among several others, and their improvements and adaptations. The main advantage of semi analytical methods such as those mentioned here is that they are applicable to a wider category of differential equations (DE), including nonlinear PDEs (NLPDE), integro-differential equations, fractional PDEs, and fuzzy PDEs. applied the ADM, as well as ADM with integrating factor (ADMIF), to symbolically solve PDEs without specifically prescribed initial data. The Elzaki transform was combined with

the projected DTM to solve the Schrödinger equation and applied the sine-Gordon Expansion Method (SGEM) and its rational form (RSGEM), which are procedures derived from the solution of the sine-Gordon (s-G) equation, to determine waveform solutions of the first of the Kadomtsev–Petviashvili hierarchy. In another modification, the DTM was coupled with the techniques of Laplace transformation and Padé approximation for solving Volterra IDE and pantograph type DEs with proportional delays.

LITERATURE REVIEW

Olotu Olanrewaju T et.al (2025) This study presents a comprehensive analysis of the Differential Transform Method (DTM) as an effective tool for solving ordinary differential equations (ODEs) of various orders. Emphasis is placed on the method's ability to handle both linear and nonlinear ODEs without the need for common simplification techniques such as linearization, discretization, or perturbation, which often introduce additional complexities or reduce accuracy. By systematically applying DTM to different classes of ODEs, the study highlights its versatility and accuracy in handling initial value problems across a range of complexities with the solution of the first, second, third and fourth-orders ODEs. Comparative analyses with analytical methods demonstrate the superiority of DTM in terms of computational efficiency and solution accuracy.

Musilimu Taiwo et.al (2024) This work presents an analytical solution of some nonlinear delay differential equations (DDEs) with variable delays. Such DDEs are difficult to treat numerically and cannot be solved by existing general-purpose

codes. A new method of steps combined with the differential transform method (DTM) is proposed as a powerful tool to solve these DDEs. This method reduces the DDEs to ordinary differential equations that are then solved by the DTM. Furthermore, we show that the solutions can be improved by Laplace–Padé resummation method. Two examples are presented to show the efficiency of the proposed technique. The main advantage of this technique is that it possesses a simple procedure based on a few straight forward steps and can be combined with any analytical method, other than the DTM, like the homotopy perturbation method.

Manan A. Maisuria et.al (2023) This study presents the mathematical model of pollutant transport in a river. To effectively find the analytical solution of the advection diffusion equation under various forms of suitable initial conditions, the reduced differential transform method (RDTM) is used. Three different initial concentration function cases, including rational, exponential, and power, are analysed for the present model. A 2D and 3D visual comparison of the solutions obtained for each case is also shown. This study discusses the sufficient condition for convergence of the reduced differential transform approach to solving non-linear differential equations. The convergence results for the concentration functions in each case are briefly described. The present method is highly effective and more efficient in solving real-world problems. For all cases, the amount of phosphate pollutant concentration at various distances and time levels has been examined using numerical and graphical representations.

H.H. Mehne et.al (2022) The complexity of solving differential equations in real-

world applications motivates researchers to extend numerical methods. Among different numerical and semi-analytical methods for solving initial and boundary value problems, the differential transform method (DTM) has received notable attention. It has developed and experienced generalizations for implementing other types of mathematical problems such as optimal control, calculus of variations, and integral equations. This review aims to provide insight into DTM. History, theoretical base, applications, computational aspects, and its revisions are reviewed. The present study helps to understand the theory, capabilities, and features of the DTM, as well as its drawbacks and limitations.

Minakshi Mohanty et.al (2021) In this work three integral transforms through modified Adomian decomposition method (ADM) are proposed to obtain the approximate analytical solution of different types of mathematical models arising in physical problems. These transformations are applied for both homogeneous and non-homogeneous linear differential equations. The efficiency and accuracy of the proposed methods are implemented through higher order non-homogeneous ordinary differential equations. Numerical tests are reported for applicability of the current scheme based on different transformations and compared with exact solutions.

Modelling Process

Modelling problems are open-ended: there are many different solutions, different levels of complexity, and different tools that can be applied. It's a challenge even to know where to start! To begin the modelling process, we need to clearly state

the problem so that we know what we are trying to solve.

Variables and Parameters

Independent variables are quantities that are input into the system and dependent variables are the quantities that are output from the system and that we are trying to predict. We should be able to identify the variables from a clearly articulated problem statement. Parameters are quantities that appear in the relationships between variables. We must list all variables and parameters in the system, give each a name and symbol and identify their dimensions such as length, mass and time.

Assumptions and Constraints

Assumptions reduce the complexity of the model and also help define relationships between variables and parameters in the system. For example, we often assume that the force of gravity is constant for an object moving near the surface of the Earth. However, we would not assume that the gravitational forces of celestial bodies are always constant. Constraints describe the values that our variables and parameters are allowed to take. For example, the mass of an object is always positive.

Build Solutions

Once we have a clear problem statement and lists of variables, parameters, assumptions and constraints, then we need to decide what mathematical tools to use to construct the model. It should be clear from the context if our model is deterministic, stochastic, data-driven or perhaps a combination. Once we have decided on the kind of model to use, we apply all the tools available.

Mathematical modelling

Models describe our beliefs about how the world functions. In mathematical modelling, we translate those beliefs into

the language of mathematics. This has many advantages

Mathematics is a very precise language. This helps us to formulate ideas and identify underlying assumptions.

- Mathematics is a concise language, with well-defined rules for manipulations.
- All the results that mathematicians have proved over hundreds of years are at our disposal.
- Computers can be used to perform numerical calculations.

There is a large element of compromise in mathematical modelling. The majority of interacting systems in the real world are far too complicated to model in their entirety. Hence the first level of compromise is to identify the most important parts of the system. These will be included in the model; the rest will be excluded. The second level of compromise concerns the amount of mathematical manipulation which is worthwhile.

Modelling model output

Evaluating complex models can often take a great deal of computer time. When the model has to be evaluated repeatedly, we may accept some loss of precision in the evaluation if it reduces the time taken per evaluation. Clearly, a reasonable approximation to the model is needed, but when we remember that the model itself is only an approximation, we will realize that a small loss of detail should be no great worry if it speeds up the calculations enough.

There are two ways to proceed. The first is to develop some approximation by simplifying, or summarizing the model mathematically. An increasingly widespread example of this is the use of so-called closure equations to summarize the

statistics (typically the mean and variance) of a stochastic model. Since closure equations are differential equations the (approximate) results of many stochastic simulations are obtained for a cost roughly equal to one deterministic model run!

The alternative approach is both simple and more general, but may run against the grain. Firstly, forget all that you know about the interacting equations in the model. Secondly, conduct an experiment in which predictions are obtained from the model for carefully chosen values of the control variables.

Estimating model parameters

Estimation of model parameters clearly comes before assessing model performance. However, we will discuss it here because it is related to the measures of performance (B, SD and MSE) discussed above. If we have a set of data D and wish to determine model parameters from this data one obvious idea is to minimise one of our performance measures with respect to the parameter values. This will give a best fit set of parameters. This also helps to explain why we should use different data for validation and model fitting, since our parameter estimation ensures a good fit to the latter. A common choice for parameter estimation is to minimise the mean square error. One perceived advantage of this is that if one assumes that errors in the data are normally distributed and uncorrelated between observations O_i then the slope of the error surface around the minimum parameter values can be used to calculate the standard errors in the parameter estimates. The problem with this is that the assumption of normality may not be valid, and often the data points cannot reasonably be considered uncorrelated (e.g. population size - or any other variable!) over time.

RESEARCH METHODOLOGY

The way we achieve this is, it uses a numerous variable to represent internal states, inputs and outputs and collection of equations and inequity to express their interplay. The use of fractional differentiation for the mathematical modelling of physical phenomena has been widespread in various connected branches of science and technology. Application of Mathematical modelling knowledge in understanding plant disease trends, develop as well as assay strategies to combat it can help us to avoid global food outage and fulfil one of the humans' basic needs. The basic and essential results related with solution for differential equations of arbitrary order may be found. The integer order derivatives are local in nature whereas the fractional derivatives are nonlocal i.e., using the integer order derivative, we can analyse the variations in neighbourhood of a point but by employing fractional derivative, we can study the changes in complete interval. It provides us extremely large freedom to choose equation type of linear sub-problems, physical parameters and initial guess. Due to this, the complicated nonlinear differential equations can often be solved in a simple way. The novelty of the proposed technique is that, it provides the large convergence region, simple solution procedure and non-local effect in the obtained solution. The future scheme controls and manipulates the series solution, which rapidly converges to the exact solution in a small acceptable region. The numerous diseases which infect vascular system of plants is regarded as wilt diseases.

RESULTS AND DISCUSSIONS

In this part, "the tabular values and figures are evaluated that is obtained by DTM and

CFDTM. Tables 1 - 6 shows the solution for $s(\tau)$, $e(\tau)$, $i(\tau)$, $a(\tau)$, $r(\tau)$ and $r^-(\tau)$ respectively obtained by DTM and CFDTM. The solution for $s(\tau)$, $e(\tau)$, $i(\tau)$, $a(\tau)$, $r(\tau)$ and $r^-(\tau)$ is DTM and CFDTM".

Table 1: Table of susceptible population $s(\tau)$ at different values of γ .

τ	$\gamma = 0.7$	$\gamma = 0.8$	$\gamma = 0.9$	$\gamma = 1$
0	6 8.0655 18×10	6 8.0655 18×10	6 8.0655 18×10	6 8.0655 18×10
0.2	6 6.033369 096×10	6 6.484589 990×10	6 6.833065 410×10	6 7.101093 027×10
0.4	6 5.122725 733×10	6 5.565943 137×10	6 5.947133 419×10	6 6.268496 109×10
0.6	6 4.563350 224×10	6 4.923185 848×10	6 5.262212 518×10	6 5.567727 247×10
0.8	6 4.230875 570×10	6 4.480399 392×10	6 4.744329 597×10	6 4.998786 440×10
1	6 4.068914 309×10	6 4.200665 954×10	6 4.375796 013×10	6 4.561673 690×10

Table 2: Table of exposed population $e(\tau)$ at different values of γ .

τ	$\gamma = 0.7$	$\gamma = 0.8$	$\gamma = 0.9$	$\gamma = 1$
0	5 2.00 000 $\times 10$	5 2.00 000 $\times 10$	5 2.000 00 $\times 10$	5 2.00000 $\times 10$
0.2	5 3.2684	5 3.0266	5 2.82474	5 2.66010967

	33550 × 10	61389 × 10	0027 × 10	7 × 10
0.4	5 3.7011 54772 × 10	5 3.4977 61938 × 10	5 3.31224 7590 × 10	5 3.14514150 8 × 10
0.6	5 3.9671 27917 × 10	5 3.7917 76166 × 10	5 3.63798 7762 × 10	5 3.49692151 5 × 10
0.8	5 4.1946 40877 × 10	5 4.0135 26485 × 10	5 3.87556 0639 × 10	5 3.75727571 0 × 10
1	5 4.4471 24079 × 10	5 4.2254 42742 × 10	5 4.07907 9382 × 10	5 3.96803011 6 × 10

Table 3: Table of infected population $i(\tau)$ at different values of γ .

τ	$\gamma = 0.7$	$\gamma = 0.8$	$\gamma = 0.9$	$\gamma = 1$
0	282	282	282	282
0.2	257.416 2224	258.964 9135	262.029 7516	265.284 0176
0.4	267.275 5013	259.687 5819	257.497 1462	257.840 4747
0.6	291.031 7094	273.293 4962	264.150 4561	259.669 3712
0.8	324.614 7581	297.093 1357	280.719 2215	270.770 7074
1	366.026 0950	329.651 0709	306.469 8571	291.144 4832

Table 4: Table of asymptotically infected population $a(\tau)$ at different values of γ

τ	$\gamma = 0.7$	$\gamma = 0.8$	$\gamma = 0.9$	$\gamma = 1$
0	200	200	200	200
0.2	175.2188 217	171.9948 872	173.6759 342	176.9707 974
0.4	210.7055 056	187.2084 738	176.6761 630	172.6846 028
0.6	272.0075 510	227.0910 139	201.6657 364	187.1414 162

0.8	351.8823 809	286.7638 747	246.3121 481	220.3412 377
1	446.7043 409	363.5699 590	309.2400 252	272.2840 671

Table 5: Table of recovered population $r(\tau)$ at different values of γ

τ	$\gamma = 0.7$	$\gamma = 0.8$	$\gamma = 0.9$	$\gamma = 1$
0	0	0	0	0
0.2	73.26341 490	56.78922 133	44.58936 555	35.23388 508
0.4	115.6931 661	92.55862 443	76.59150 701	64.52546 644
0.6	162.5740 800	129.0616 812	107.6391 594	92.47760 532
0.8	220.9500 804	173.2748 196	143.7130 733	123.6931 630
1	294.4142 649	229.7014 251	189.5803 826	162.7750 005

Table 6: Table of reservoir population $r^-(\tau)$ at different values of γ

τ	$\gamma = 0.7$	$\gamma = 0.8$	$\gamma = 0.9$	$\gamma = 1$
0	50000	50000	50000	50000
0.2	49769.14 447	49827.93 009	49869.73 307	49900.15 898
0.4	49625.51 790	49700.78 918	49757.18 422	49800.51 375
0.6	49503.24 037	49586.62 441	49650.62 486	49701.06 843
0.8	49393.12 443	49480.22 435	49547.85 429	49601.82 712
1	49291.29 976	49379.30 131	49447.86 689	49502.79 392

the Differential Transform Method and Conformable Fractional Differential Transformation Method has been successfully applied to find the solution of nonlinear system of ordinary differential equation in COVID-19. A series solution is obtained as a result of DTM and CFDTM for the introduced classes of nonlinear differential equation. Further, the behaviour of the outcomes is discussed in plots. Also,

the effectiveness of preventive measures, predicting future outbreaks and potential control strategies of the solution obtained are discussed”.

CONCLUSIONS

The Fractional Differential Transform Method takes the methodology and applies it to fractional-order differential equations; this method is now widely used to represent anomalous diffusion, memory-dependent processes, and viscoelastic materials. The development of hybrid techniques that combine DTM with numerical discretization processes such as finite differences, Adomian decomposition, or Laplace transforms has also allowed for the handling of increasingly complicated models. The research community's dedication to expanding DTM's capabilities and embracing its inherent flexibility is evident in these advances. As mathematical modeling expands into new fields, hybrid techniques like DTM's enable researchers in fields as diverse as bio-mathematics, ecology, epidemiology, and nanotechnology to take on novel issues. Throughout the paper, the computational simplicity of the DTM is cited as a key asset compared to other semi-analytical approaches. The development of correction functionals or embedding parameters is not necessary, unlike the Variational Iteration Method or the Homotopy Analysis Method, and unlike perturbation techniques, it does not need the presence of a small parameter. This not only makes the method easier to understand and apply, but it also makes it accessible to researchers without a strong foundation in advanced mathematics. Maple, MATLAB, and Wolfram Mathematica are symbolic computing applications that work well with DTM because to its algorithmic nature.

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