MATHEMATICAL ANALYSIS OF SYSTEM RELIABILITY USING MARKOV CHAINS

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ABSTRACT

In contrast, the solution of the mathematical models developed in this study requires relatively insignificant computer time while achieving high prediction accuracies. This facilitates the need for software tools such as Relex Software to handle the systems' reliability. Many engineering systems are subject to failure after a given amount of time, and just how long a system will not fail depends on its design. Thus, a client of the Statistical Consulting Collaboratory requested an analysis of the reliability of his telecommunications system. One very useful useful technique in finding the reliability of a system is Markov modeling. This technique involves analytically finding the probability of the system being in each of its potential states, and when summed appropriately these state probabilities lead to the overall reliability of a system. However, for systems with a large number of states, solving for the reliability becomes increasingly tedious and complex. The most common method currently utilized in practice for handling the reliability predictions of systems having components with non constant failure rates are based on Monte Carlo Simulations. However, to obtain the required high accuracies for moderately complex system reliability models, Monte Carlo Simulations may require excessive computer time.

Keywords: *Markov chain, probability, Markov modeling, Monte Carlo, Relex Software.*

INTRODUCTION

Reliability is the probability of a device performing its purpose adequately for the period intended under the given operating conditions. In Reliability models, the states usually represent the various working and failed conditions of the system. Markov Processes are widely used in Engineering, Science and Business Modeling. Markov Modeling is a widely used technique in the study of Reliability analysis of system. They are used to model systems that have a limited memory of their past. In a Markov Process, if the present state of the process is given, the future state is independent of the past. This property is usually referred to as the Markov Process. Markov modeling is a modeling technique that is widely useful for the Reliability analysis of complex systems. It is very flexible in the type of systems and system behavior it can model. This modeling technique is very helpful in most of the situations .The model is quite useful to modeling operation system with dependent failure and repair models. In fact it is widely used to perform Reliability and Availability analysis of responsible system with constant failure and repair rates. From time to time the Markov method is also used to perform human Reliability Analysis. Reliability is defined as the probability that a system or component will function over intended time period. Markov model is a technique that works well when failure hazards and repair hazards are constant. Markov process involves different states. The fundamental assumption in a markov process is that the transition probability from i to j and is independent of all previous states. Markov models are widely used in Reliability and Availability. Discuss the markov model in system reliability with applications.

LITERATURE REVIEW

Juan A. Cruz-Juárez (2023) A nonhomogeneous Markov chain is applied to the study of the air quality classification in Mexico City when the so-called criterion pollutants are used. We consider the indices associated with air quality using two regulations where different ways of classification are taken into account. Parameters of the model are the initial and transition probabilities of the chain. They are estimated under the Bayesian point of view through samples generated directly from the corresponding posterior distributions. Using the estimated parameters, the probability of having an air quality index in a given hour of the day is obtained.

Christian Kwaku Amuzuvi (2022) Operators of renewable energy systems (RESs) must always manage uncertainty to some extent to ensure the reliability and the security of the electric power supply source. The guiding principle in this regard is to ensure service reliability and quality by balancing load variations with the variable renewable energy (VRE) sources. The WEGS and the varying load were modelled separately after which the two were combined into one model. Full availability, partial availability, the expected energy not supplied (EENS) or loss of energy expectation (LOEE), the mean or average instantaneous electric power generation and mean instantaneous generation deficiency were the indices used for the evaluation of the WEGS.

Naima Tamaloussi (2020) In the industrial sector, maintenance of production facilities plays an important role to carry out production by increasing the reliability and availability of the production process. Predictive maintenance strategy seems adequate to anticipate the failure and degradation of the state of such equipment. In this study, we present the results of a stochastic modeling conducted on the analysis of the availability of motor-pump system, installed in a cooling circuit in an industrial complex. The proposed model is a dynamic Markovian approach, for the purpose of a comparison with the analytical calculation in terms of the indicators' evaluation of the dependability of the studied system, including instant availability.

Esteban Flores-Méndez (2020) The damage accumulation mechanism is unit jump type, depending on the state. It uses a shock model based on Bernoulli trials and probabilities to remain in the same state or the next one. Data are adjusted to Lognormal distribution and proven with a Kolmogórov-Smirnov test. The vector obtained from multiplying the initial state vector with the transition matrix was developed and the system of equations to find each transition probability with a single inspection report was solved. In order to calculate propagation of internal corrosion after inspection, an exponential equation was proposed and a parameter was adjusted to the data.

T. Sumathi Uma Maheswari (2019) In this study probability of n component system with m number of components failed with different failure rates has been derived by markovian approach and also obtained the reliability expressions for n component system which is to be worked with at least k number of components should work.

Markov chain

A Markov chain or Markov process is a stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event. Informally, this may be thought of as, "What happens next depends only on the state of affairs now." A countably infinite sequence, in which the chain moves state at discrete time steps, gives a discrete-time Markov chain (DTMC). A continuous-time process is called a continuous-time Markov chain (CTMC). It is named after the Russian mathematician Andrey Markov.

Markov chains have many applications as statistical models of real-world processes, such as studying cruise control systems in motor vehicles, queues or lines of customers arriving at an airport, currency exchange rates and animal population dynamics.

Applications of Markov Chains

One common application of Markov chains is in the field of economics, where they can be used to model the behaviour of stock prices. In this case, the possible states of the system might represent upward or downward trends in the market, and the transitions between states might be influenced by various economic factors such as interest rates and the performance of individual stocks. By analyzing the probability of transitioning between different states, it is possible to make predictions about the future behaviour of the market. Markov chains can be used for modeling biological systems, such as the spread of a disease through a population. In such case, the states of the system might represent the number of individuals in a population who are susceptible, infected, or immune to the disease, and the transitions between states might be influenced by several factors such as the rate of infection and the effectiveness of a vaccine. By analyzing the probability of transitioning between different states, it is possible to make predictions about the future spread of the disease and the effectiveness of various control measures.

Markov model

In probability theory, a Markov model is a stochastic model used to model pseudo-randomly changing systems. It is assumed that future states depend only on the current state, not on the events that occurred before it (that is, it assumes the Markov property). Generally, this assumption enables reasoning and computation with the model that would otherwise be intractable. For this reason, in the fields of predictive modelling and probabilistic forecasting, it is desirable for a given model to exhibit the Markov property.

Markov-chain forecasting models

Markov-chains have been used as a forecasting method for several topics, for example price trends, wind power and solar irradiance. The Markov-chain forecasting models utilize a variety of different settings, from discretizing the time-series to hidden Markov-models combined with wavelets and the Markov-chain mixture distribution model (MCM).

Markov chain Monte Carlo

In statistics, Markov chain Monte Carlo (MCMC) is a class of algorithms used to draw samples from a probability distribution. Given a probability distribution, one can construct a Markov chain whose elements' distribution approximates it – that is, the Markov chain's equilibrium distribution matches the target distribution. The more steps that are included, the more closely the distribution of the sample matches the actual desired distribution. Markov chain Monte Carlo methods are used to study probability distributions that are too complex or too highly dimensional to study with analytic techniques alone.

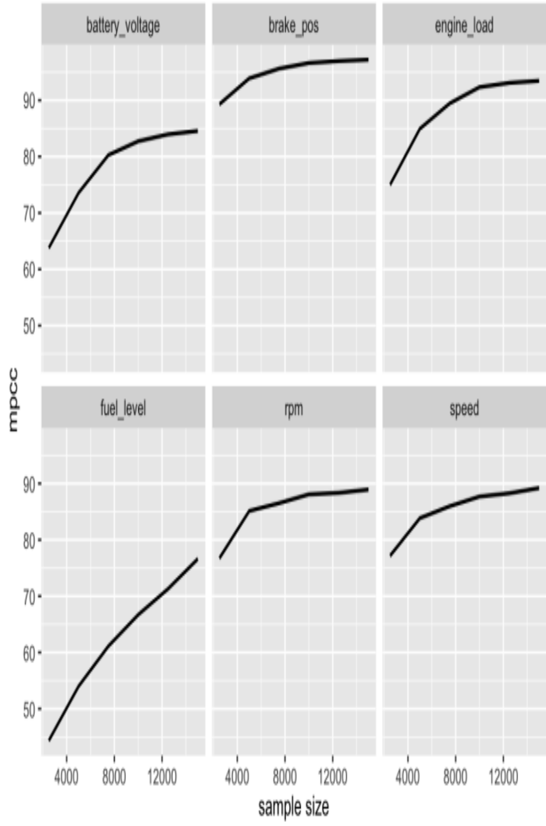
Various algorithms exist for constructing such Markov chains, including the Metropolis–Hastings algorithm.

RESEARCH METHODOLOGY

The steady state does not mean that the Markov Chain has stopped changing but it only explains that the probabilities of the model will not change with respect to time after a given period. In the two-state condition as modelled, it was observed that the probability matrix had the characteristics of a mixed mode loading, whereby it changed from an independent loading probability state condition to a dependent loading probability state. It was observed that the Markov Chain displayed properties of generating new but random sequence data for the sequences of maximum and minimum load conditions that were almost similar to the field data. The loading condition of the crankshaft is mathematically modelled in the conditional probability method. The fatigue failure of the crankshaft is considered to be stochastic in nature, where the failure occurs at a given loading vector interval, randomness of the loading vector intervals, uncertainties in the crankshaft model, such as the geometric parameter and material properties, rainflow analysis to replicate the variable amplitude loading experience by the crankshaft, manufacturing defaults due to the ability of the Markov Chain to generate a new sequence of numerically random, but almost similar, data to the original sequence, as has been illustrated extensively by the relevant researchers. Thus, the first order discrete Markov Chain was used to generate this sequence as mentioned earlier, and would be further extended by the reliability assessment through the hazard rate (λ) bathtub curve.

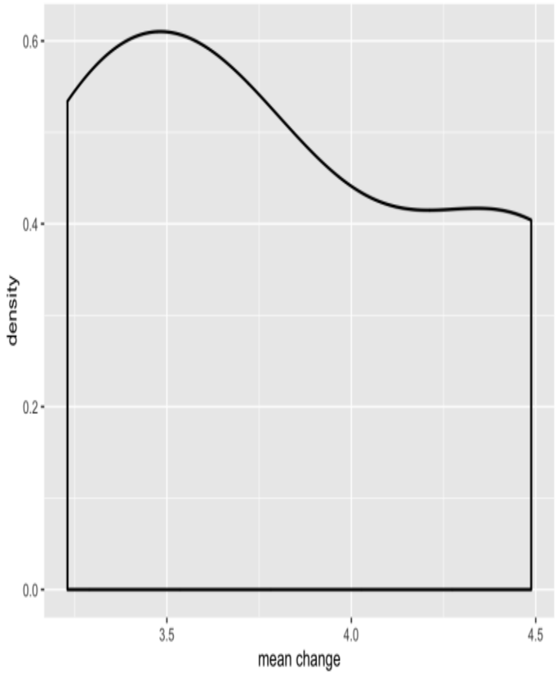
RESULTS AND DISCUSSIONS

When analyzing the effect of sample size on PCC, the results of the five vehicles were averaged, which included the various states tested. It was concluded that larger sample sizes needed to be tested as an increase in the mean PCC was observed for all attributes of interest. Graph 1 provides a plot of the results. It was then established that the sample size would need to be at least 15,000 observations.



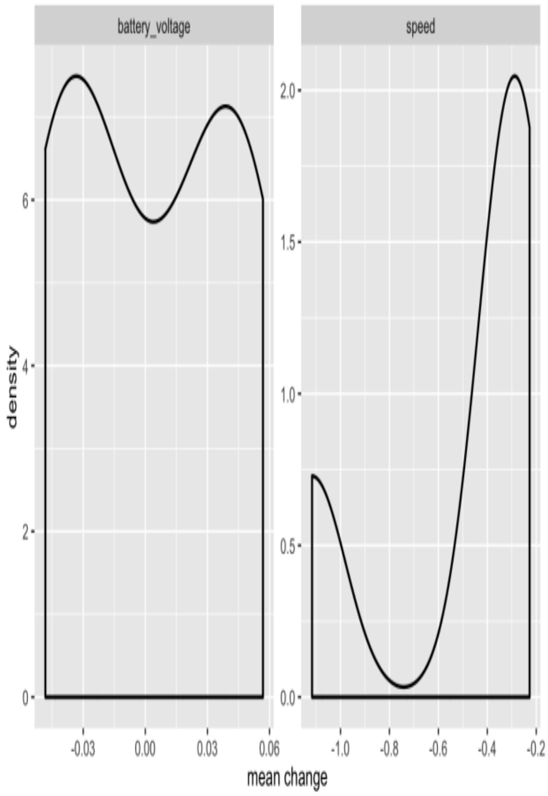
Graph 1: Average of the vehicles for the means PCC

The analysis of the inconclusive results began by reevaluating the bin method for speed. It was expected that there would be different results as the sample size is now restricted to 15,000 observations. Graph 2 shows the plot of the test results.



Graph 2: Distribution of the change in mean for the bin method associated with speed

As a positive value indicates that the range method is superior, it was concluded that the range method is superior for speed. Next, the model type for battery voltage and speed was examined. The test was conducted as before which resulted in Graph 3.



Graph 3: Distribution of the change in mean for the model type of speed & battery voltage

As a negative value indicates that the multivariate model increased the PCC, it was concluded that the multivariate model is superior for speed and the result was still inconclusive for battery voltage. A significant difference between the multivariate and univariate model for battery voltage could not be determined, thus the univariate model for battery voltage was used to avoid over fitting.

Table 1: Second Fractional Factorial Design

Factor	#of Levels	Description
Number of States	5	3|6|9|12|15
Attribute*	6	rpm|Speed|Engine Load|Brake Position|Battery Voltage Fuel level|

		Transmission Gear
Sample Size	8	15000|20000|25000|30000|35000|40000|45000|50,000

*Transmission gear was the conditional variable for the multivariate Markov Chain.

As the bin method and the model type had been identified for each attribute, these factors were fixed and the effect of sample size and the number of states were explored for a broader range.

CONCLUSION

Probability of n component system with m number of components with different failure rates has been derived by markovian approach and also derived the reliability expressions for n component system which is to be worked with at least n-k number of components existed to work. This method incorporates differential equations into the solving of Reliability, Availability, steady state and time dependent probabilities to the various states in a Markov model. Markov analyses' challenges are that the differential equations become very difficult to solve by hand when the systems become large and complex, such as real systems today. Markov chains are a useful tool for modeling and analysing systems with a finite number of states and transitions among those states. They have many applications in fields such as economics, biology, and computer science, and can be represented using transition matrices or directed graphs. By analysing the probabilities of transitioning between different states, it is possible to make predictions about the long-term behaviour of the system and to understand the underlying dynamics. The study has thus provided an accurate and practical method for solving complex system reliability modeling problems. Present reliability models in the most part are based on constant failure rates wherein the probability of a component failure remains independent of the past history of the component operation. Therefore, it is necessary to use powerful software tools, such as Relex software, to solve larger Markov models.

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