AN OVERVIEW OF THE ANCIENT INDIAN MATHEMATICAL SYSTEMS

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Abstract

This article aims to provide an overview of the mathematics that was practiced in ancient India along with a discussion of pertinent sources and possible directions for further study. Because of this, it might be beneficial to group the information into the following more broadly based categories: Vedic mathematics, Jaina mathematics, the evolution of the number system and numerals, and the mathematical astronomy tradition.

Vedic mathematics

The Sulvasutras contributed significantly to the mathematical corpus that has survived from the Vedic period. The Sulvasutras are compositions that provide instructions for the construction of the vedis (altars) and agnis (fireplaces) required for the performance of yajnas, which were fundamental to Vedic culture. The Sulvasutras describes the construction of such platforms with tiles of moderate dimensions, straightforward shapes such as squares and triangles, and occasionally unique shapes such as pentagons. Depending on the context and purpose of the particular yajna, the fireplaces were constructed in a variety of shapes, such as falcons, tortoises, chariot wheels, circular basins with handles, pyres, etc. Several of the vedis involved, particularly for the yajnas for special occasions, had proportions of the order of 50 to 100 feet, therefore sketching perpendiculars in that environment was necessary while constructing the overall layout. They were aware of the "Pythagoras theorem," and all four main Sulvasutras mention it explicitly. This was achieved by employing both the method currently taught in schools, involving the perpendicularity of the line joining the centers of two intersecting circles, and the line joining the two points of intersection. The Sulvasutras also describe a number of geometrical constructs and ideas, such as how to convert a square into a circle with an equal area and vice versa, as well as a close approximation to the square root of two (for more information, see [4]). As with other Vedic knowledge, the Sulvasutras were only ever transmitted orally over an extended period of time. Several Sulvasutras have also been accompanied by Sanskrit commentaries, but it is uncertain when they were composed. It is unknown when the first written variants of the Sulvasutras appeared. Beginning in the second half of the nineteenth century, academicians in Europe began publishing text transcripts alongside contemporary commentary. Regarding the origins of his study of the Sulvasutras, Thibaut observes that Mr. A.C. Burnell was the first to recognize the Sulvasutras' significance. On page 29 of his Catalogue of a Collection of Sanscrit Manuscripts, he stated, "We must turn to the Sulva portions of the Kalpasutras for the origins of the Brahman. While the present translations are quite exhaustive, there are still significant voids, particularly in the case of the Manava Sulvasutra, which is more concise than the others. R.G. Gupta [13], Takao Hayashi, and the present author, as well as possibly others unknown to the original translators, have all disclosed new information. Insufficient mathematical knowledge among the translators may be one of the contributing factors in this regard. To provide a solid foundation for the mathematical information in the Sulvasutras, a comprehensive review is required. Moreover, there is space for development in the interrelationships of the numerous Sulvasutras' outcomes.

The ceremonial framework of the Sulvasutras lends itself to the question of bridging the ritual and mathematical components, as well as integrating with other comparable circumstances in other civilizations; Seidenberg [36] offers a perspective on this. Can any of the fireplaces from earlier periods be located? is another logical argument that arises in the context of the Sulvasutras. It appears from the description of the masonry structure that they would not have been able to withstand the elements for very long; however, it must be remembered that the mission did not require a durable structure. Nevertheless, during excavations at Singhol in Punjab, a substantial masonry structure in the conventional form of an avian with extended wings was discovered. It is dated to the second century BCE ([18], p. 79–80 and [25], footnote on page 18), but its numerical specifications differ significantly from those mentioned in the Sulvasutras.

Finding alternative sites is still a possibility, but is unlikely to be particularly fruitful. In addition to the Sulvasutras, mathematical studies of the Vedas with an emphasis on numerical comprehension have been conducted. The Rigveda demonstrates a remarkable preoccupation with numbers for a work with such a broad scope, encompassing both the spiritual and the secular; see [2] for more information. (It should be noted, however, that the numbers were not recorded; the focus here is on number titles.) The Yajurveda provides terms for powers of 10 up to 1012, and numerous fundamental mathematical properties, such as [25], are discovered to be applicable in various contexts. There is still space for development in our overall understanding of evolution, which requires knowledge of both mathematics and Vedic sanskrit.

Development of the number system and numerals

Buddhism, Jainism, and Veda have all been examined in relation to the development of India's numeral system. As stated in the previous sections on the Vedic and Jaina traditions, India has always been fascinated by extremely large numbers. Tallakshana, a Buddhist term from the past, represented the number 1053, and Buddha was renowned for his proficiency with numbers. Also mentioned in the Buddhist tradition are large quantities. However, the names of the powers of ten varied according to tradition and era; for example, Parardha, which literally translates to "halfway to heaven," represented 1012 in early literature but 1017 in later works such as those of Bhaskara II. It is believed that the oral practice of utilizing various powers of ten was a significant factor in the subsequent development of decimal representation in writing, which appears to have occurred in the first centuries of the common era. This connection is not entirely apparent, however, due to a period of several hundred years during which written forms of numbers did not correspond to place value notation (see below). Moreover, the other systems appear to have been in use for a considerable period of time, even after the decimal place value system with zero acquired popularity. The apparent simplicity and elegance of the decimal place value system raises queries as to why the oral tradition frequently refers to powers of 10, given the prevalence of such references. Despite this, it appears that the Chinese began using the decimal place value system to represent numerals as early as the third century BCE. They omitted the zero symbol and substituted a vacant space in its place. Regarding whole numbers, the introduction of zero as a substitute paved the way for modern number writing. The current system of comprehensive decimal representation, which includes fractions and a decimal point, originated in Europe in the 15th century, although it is believed that the Arabs used it first in the 10th century. Early common period India conceptualized zero as a number and incorporated it into the number system. In Brahmagupta's Brahmasphut asiddhanta, written in the seventh century, there is a systematic presentation of negative number arithmetic. The development of the numerals is a relevant topic.

Numerous written numerals have been investigated. The earliest of these would have been Indus seals with numerals represented by the strokes. Among the ancient numerals are the Kharosthi numerals, which were used between the third century BCE and the third century CE and can be found in the inscriptions from Kalderra, Taksasila, and Lorian, as well as the Brahmi numerals from Naneghat (first century BCE), which did not use the place value system. However, R. Saloman argues in [32] that this inscription is a forgery. The earliest zero in an Indian inscription dates back to 876 CE and was discovered in a temple near Gwalior (credit to Bill Casselman, an image of this can be viewed online).

In the late nineteenth century, Bhagwanlal Indra ji conducted a great deal of research, which is described in George Ifrah's book [16]; (although much of what Ifrah says has been contradicted by various reviewers - see [8] for details - one may assume that his account of Bhagwanlal Indra ji's work is accurate). In addition to the stone inscriptions, the presence of decimal numbers has been examined on copper plates that served as legal documents from the seventh to the tenth centuries and recorded grants of donations by monarchs or affluent individuals to Brahmanas. Due to attempts to misappropriate the properties involved, there have been objections to this source due to the plates' susceptibility to forgery; however, while this may apply to a few plates, the plates as a whole should not be discounted as a source; for a discussion on this, see [7], pages 44–48, where the author attempts to refute the objections. In this respect, another source are the numerals discovered in ancient texts. Numerous mathematical traditions employ large numbers; therefore, it would be intriguing to examine how these numbers are represented in manuscripts from various periods. The origin of these systems would also be a related issue, given that various Indian languages have their own unique numeral symbols. The author is unaware of any comprehensive research on the topic. This content should be archived systematically from a variety of sources, and then the sources should be analyzed to ascertain the development of the ideas.

Mathematics from the Jaina tradition

The Jainas have a lengthy tradition of employing mathematics in daily life. They were motivated by thought as opposed to rituals, which they despised, and they had a sophisticated understanding of the world. According to Jaina cosmology, the universe is a flat plane with concentric annular regions encompassing the Jambudvipa (island of Jambu), an innermost circular region with a diameter of 100,000 yojanas. The annular regions consist alternately of water and land, their widths doubling with each succeeding ring. It should be noted that this cosmology is also present in the Puranas. Although most of the researchers were philosophers and not mathematicians, the geometry of the circle played a significant role in the discussion. In Suryaprajnapti, which is believed to date to the fourth or fifth century BCE (the earliest extant manuscript dates to around 1500, on paper), and in the writings of Umasvati, who is believed to have lived in the second century CE according to the Digambara tradition of the Jainas and in the first century BCE according to the Svetambara tradition, numerous properties of the circle are described.

Suryaprajnapti cites the then-conventional number of three for it, but rejects it in favor of ten. This departure from the ancient concept of 3 as the ratio of circumference to diameter is one of the distinguishing characteristics of the Jaina tradition. The Jainas were also aware that the ratio of the circumference to the diameter and the area to the square of a circle's circumference and diameter are identical. In addition, they had compelling estimates for the lengths of circular arcs, the areas they occupied, and the corresponding chord.

It is unclear how these formulations were derived. In addition to permutations and combinations, sequences, and the categorization of infinities, Jaina literature also explores other mathematical topics in depth, such as sequences and categorization of infinities. There is definite evidence of mathematical activity in the Jaina tradition beginning in the eighth century, and it may have persisted until the middle of the fourteenth century. The Ganita-sarasangraha by Mahavira, composed in 850, is one of the most well-known works in this regard. Virasena (eighth century), Sridhara (between 850 and 950), Nemicandra (approximately 980 CE), and T. hakkura Pheru (fourteenth century) are additional figures that may be mentioned in relation to the evolution of mathematics in the Jaina canon.

In the works of Thakkura Pheru from the early 14th century, one can discern a merger of indigenous Jaina heritage and Indo-Persian literature. A portion of the geometry described, involving domes, arches, and other structures, is closely associated with the development of Islamic architecture in India. In the late 14th century, Mahendra Suri, a Jaina astronomer who served at the court of a Tughluq dynasty monarch, wrote about the astrolabe. As described in [26], Jaina astronomical writings have a number of connections to Sanskrit and Islamic science. Rangacharya published an English translation of the Ganita-sarasam graha text in 1912, and it was recently reissued [31].

In more recent times, Padmavathamma has published an edition that incorporates the original text along with translations into English and Kannada [22]. Thakkura Pheru's Ganita-sara-kaumudi was recently published due to a collaborative endeavor [34]. In [14] are a series of articles describing numerous aspects of Jaina mathematics, and [5] contains an earlier discussion of the topic. The systematic examination of Jaina texts from a mathematical standpoint has not been conducted in sufficient depth. There is a significant lack of knowledge regarding ancient Jaina writings from the centuries BCE to the early centuries CE. The initial stage is to collect sufficient information on the available resources.

The mathematical astronomy tradition

Indian mathematics has been dominated by the Siddhanta or mathematical astronomy tradition since the third or fourth century, when it began to flourish. This inheritance has remained essentially constant. Aryabhata (476–550), the first significant figure in the tradition, is credited with establishing scientific astronomy in India. Despite the fact that Bhaskara II (1114–1185) is considered to be the last prominent proponent of the Siddhanta tradition, it continued after his time.

The Aryabhatiya, which was composed in 499 and is the eldest complete Siddhanta text still extant, is fundamental to both the tradition and the subsequent works of the Kerala school of Madhava, which I will examine below. The Gitikapada, Ganitapada, Kalakriyapada, and Golapada chapters contain a total of 121 verses. In a single stanza, the first describes cosmology and provides a table of 24 sine differences separated by 225 minute arc intervals. As its title suggests, the second chapter is devoted to mathematics and contains, among other things, formulas for finding square and cube roots, a rough expression for, formulas for the areas and volumes of various geometric figures, formulas for the sums of consecutive integers, formulas for the sums of squares, formulas for the sums of cubes, and formulas for computing interest. For additional information, please refer to [40] and [41]. The next two chapters cover astronomy and topics such as eclipses, planet distances, and relative motions, etc. (details will not be provided here).

Varahamihira, Bhaskara, Brahmagupta, Govindaswami, Sankaranarayana Aryabhat, Vijayanandi, Sripati, and Brahmadeva were among the most influential figures during the time period preceding Bhaskara,Narayana. E'sa may have received the names Pandit and Gan directly from the oral tradition in later years. It should be noted that many of the dates listed here are approximations, as they are based on a number of indirect assumptions and lack of reliable historical data.The preponderance of Brahmagupta's Brahmasphutasiddhanta concerns astronomy. General mathematics is covered in the 12th and 18th chapters. In addition, the 21st chapter contains poems about trigonometry, which in Siddhanta astronomy literature was traditionally combined with astronomy. Chapter 11, which critiques earlier works such as Aryabhatiya, is another distinctive feature of the text. This tradition, like other scientific organizations, had many internal disagreements; however, the severe language used in this chapter would be disconcerting to modern readers.

Chapter 12 contains a sytematic description of arithmetic operations, including those involving negative integers, which was unknown to European mathematics until the second century's middle. The chapter also discusses geometry, with an emphasis on his famous formula for the area of a quadrilateral, which generalizes Heron's formula for the area of a triangle. However, this formula does not require the cyclicity of the quadrilateral, which was criticized by later mathematicians in the tradition. The preponderance of Brahmagupta's Brahmasphutasiddhanta can be found in the 18th chapter, which focuses on astronomy. General mathematics is covered in the 12th and 18th chapters. In addition, the 21st chapter contains poems about trigonometry, which in Siddhanta astronomy literature was traditionally combined with astronomy.

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In this regard, the cataloging and accessibility of extant manuscripts is also a crucial duty. The role and proportion of mathematics in the body as a whole appear difficult to ascertain and require extensive research. In [20], it is noted that only a small percentage of the enumerated manuscripts from the census pertain explicitly to mathematics - the census includes all manuscripts with some coherence in the exact sciences. This is a suitable starting point. In 2002, K.V. Sarma published a comparable compilation of Sanskrit literature found in the archives of Tamil Nadu and Kerala. 1966 [38] saw the publication of a bibliography of Sanskrit literature on mathematics and astronomy by S.N. Sen, A.K. Bag, and R.S. Sarma. Understanding the instruments used in astrology, such as timepieces, astrolabes, and other devices, as well as the mathematics underlying them, is an issue that must be addressed. S.R. Sarma has exerted considerable effort in this regard. Although it is commonly believed that mathematical astronomy has eclipsed the study of astronomy in India, observational and mathematical components must also be studied.

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