AN EXAMINATION OF FUZZY TOPOLOGICAL SPACES

Akash Gadge
Research Scholar
Department of Mathematics
Kalinga University, Naya Raipur,
Chhattisgarh.
gadge.akash87@gmail.com

Dr. Rahul Dwivedi
Associate Professor
Department of Mathematics
Kalinga University, Naya Raipur,
Chhattisgarh.

ABSTRACT

In the world of engineering and technology, the most significant challenges come in the form of doubt and haziness.The use of fuzzy topology may be utilized to successfully find answers to such issues. Researchers investigate fuzzy topological concepts by using the appropriate complement functions in their work. The fuzzy topological ideas are able to be hypothesized with the help of the various complement functions that are found in the fuzzy literature.This article's focus will be on investigating some of the fundamental ideas behind fuzzy topological spaces.

***KEYWORDS:** Fuzzy topological space (FTS), fuzzy continuous map, fuzzy semi-compact spaces, fuzzy semi-connectedness, fuzzy weakly-compact spaces.*

INTRODUCTION

The fields of geometry and analysis are where topology got its start. When seen through the lens of geometry, topology is the study of qualities that are maintained by a certain set of transformations known as homeomorphisms. In the study of real or complex functions, several concepts of topology may be thought of as abstractions of more traditional ideas. Open sets, continuity, connectivity, compactness, and metric spaces are all ideas that fall under this category. Prior to their generalization in topology, they constituted a fundamental component of analysis.

FUZZY TOPOLOGICAL SPACES

A topological space is an ordered pair (X, τ), where X is a set and τ is a collection of subsets of X, satisfying the axioms:-

(i)The empty set and X itself belongs to τ

(ii) Any(finite or infinite) union of members of τ still belongs to τ

(iii) The intersection of any finite number of members of τ still belongs to τ. A fuzzy topology on a set *X* is a collection δ of fuzzy sets in X such that: (i) 0,1□δ (ii) μ, □ □ δ □ μ□□ □ δ

(iv) ∀ (μ_i)□∈□□ δ□ □$_{i\in I}(\mu_i)$□□□ δ is called as fuzzy topology for X, and the pair (X, δ) is a fuzzy topological space, or FTS in short. Every member of δ is called a T-open fuzzy set.

Fuzzy sets of the form 1 − μ, where μ is an open fuzzy set, are called closed fuzzy sets. Few examples of fuzzy topologies are :-

- Any topology on a set *X* (subsets are identified with their characteristic functions), The indiscrete fuzzy topology {0, 1} on a set *X* (= indiscrete topology on *X*),
- The discrete fuzzy topology on *X* containing all fuzzy sets in *X* ,
- The collection of all crisp fuzzy sets in *X* (= discrete topology on *X*),
- The collection of all constant fuzzy sets in *X* ,The intersections of any family of fuzzy topologies on a set X.

BASE AND SUBBASE FOR FTS

A base for a fuzzy topological space (*X* ,□) is a sub collection □ of τ such that each member A of τ can be written as *V*

j□A A_j , where each A_j □□. A subbase for a fuzzy topological space (X , τ) is a sub collection S of τ such that the collection of infimum of finite subfamilies of S forms a base for (X , τ) . Let (X , τ) be an FTS. Suppose A is any subset of X. Then (A,□$_A$) is called a fuzzy subspace of(X , τ) ,Where,□$_A$ □ {B_A : B□□}, B□{(x,□$_B$(x)): x□X,B_A□{(x,□$_{B/A}$(x)): x□A}.

FUZZY POINT

CLOSURE AND INTERIOR OF FUZZY SETS

The closure A' and the interior A^0 of a fuzzy set A of X are defined as

A' □ inf{K : A □K, K^c□□}

A^0□ sup{O : O □A, O□□}

FUZZY CONTINUOUS MAP

Given fuzzy topological space and , a function is fuzzy a function f : X□Y is fuzzy continuous if theinverse image under f of any open fuzzy set in Y is an open fuzzy set in X ; that is if $f^{□1}$(□)□□ whenever □□□□

GENERALIZED LOCALLY CLOSED SETS AND GLC-CONTINUOUSFUNC

Fuzzy G-Closed sets : □∈ (□, □) is Fuzzy G-closed, ⇔□□(□) □ □,□ □ □,□ is open in (X , □). Fuzzy G- open Sets : □∈ (X , □) is fuzzy G-open,⇔ (□ − □) is fuzzy g-closed. Fuzzy Locally Closed sets :∈ is fuzzy locally closed ⇔□ = □ ∩ □,Where, □∈□ and □ is closed in (X, □). Fuzzy G-Locally closed sets:∈ (□, □) is fuzzy G-locally closed ⇔□ = □ ∩ □, Where, □ is fuzzy g-open in (□, □).Fuzzy Generalized Locally Closed Functions: Fuzzy GLC-irresolute: □: (X, □) □(□, □) ⇔ $□^{-1}$(□) ∈ □□□ (X, □) ∀ □ ∈ □□□ (□, □). Fuzzy GLC-continuous: □: (X, □) □(□, □) ⇔ $□^{-1}$(□) ∈ □□□ (X, □) ∀ □ ∈□ [1].

FUZZY SEMI-COMPACT SPACES

A FTS X is said to be a fuzzy semi-compact space if every fuzzy cover of X by fuzzy semi-open sets (such a cover will be called a fuzzy semi-open cover of X) has a finite sub- cover.A direct consequence of the above definition yields the following alternative formulation of a fuzzy semi-compact space.

FUZZY SEMI-COMPACT SETS

A fuzzy set A in a FTS X is said to be a:

(i) fuzzy compact set, if every fuzzy open cover of A has a finite sub-cover for A .

(ii) fuzzy nearly compact set, if every fuzzy regular open cover of A has a finite sub-cover forA .

(iii)fuzzy s-closed set, if every fuzzy semi-open cover of A has a semi-proximate sub- cover forA .

(iv)fuzzy almost compact set, if every fuzzy open cover of A has a finite proximate sub- cover for A.

(v)fuzzy□-rigid set, if for every fuzzy open cover U of A , there exists a finite subfamily U_0 of U such that A □ int cl(□U_0).

(vi) fuzzy $□^*$ -rigid, if for every semi-open cover U of A , there exists a finite subfamily U_0 of U such that A □ scl(□{$sclU$: U □U_0 }).

SEMI*-CONNECTEDNESS IN FUZZY TOPOLOGICAL SPACES

Let A bea subset of a fuzzy topologicalspace X . The generalized closure of A is defined as the intersection of all g-closed sets containing A and is denoted by Cl *(A) . A subset B of a fuzzy topological space X is called g-closed, if Cl(B) □ Uwhenever$B$ □ $U$and U is open in X .A subset A of a fuzzy topological space □ is called semi*-open if A □ Cl^* (Int(A)).A subset A of a fuzzy

topological space □ is called semi*-regular ifnit is both semi*-open and semi*closed. Let A be a subset of X .
Then the semi*-closure of A is defined as the intersection of all semi*-closed sets containing A and is denoted by s * Cl(A) .A subset A of a fuzzy topological spaces X , the semi*-frontier of A is defined by *s * Fr(A)* □*s * Cl(A)* \ *s * Int(A)*.A function f : X□Y is said to be
(i) semi*-continuous if *f* □1(*V*)is semi*-open in X for every open set V in Y .
(ii) semi*-irresolute if *f* □1(*V*)is semi*-open in X for every semi*- open set V in Y. A fuzzy topological space X is said to be semi*-connected if X cannot be expressed as the union ofntwo disjoint non-empty semi*-open sets in X.

FUZZY WEAKLY-COMPACT SPACES

A fuzzy subset S is said to be fuzzy regular open (resp. fuzzy regular closed) ifnint(cl(S))= S (resp. cl(int(S))=S). A fuzzy open cover {*V*□:□□*L*} of an FTS is said to be fuzzy regular if for each □□*L* there exists a nonempty fuzzy regular closed set *F*□ in *X*such that *F*□ □*V*□and *X* = U{int (*F*□):□□*L*}. An FTS X is said to be fuzzy weakly-compact (resp. fuzzy alomost- compact) if every fuzzy regular (resp. fuzzy open) cover of X has a finite subfamily whose fuzzy closures cover X. It is clear that every fuzzy almost-compat space is fuzzy weakly-compact.

A fuzzy subset S of the fts X is said to be fuzzy weakly- compact if S is fuzzy weakly- compact as a fuzzy subspace of X. A fuzzy subset S of an fts X is said to be fuzzy weakly-compact relative to X if for each cover {*V*□: :□□*L*}of S by fuzzy open sets of X satisfying the condition:- For each □□*L*, there exists a nonempty fuzzy regular closed set *F*□ of X such that *F*□ □*V*□ and *S* □ U{int (*F*□):□□*L*}, there exists a finite subset *Lo of L* such that *S*□{cl(*V*□):□□*Lo*}. An FTS X is said to be fuzzy nearly compact if every regular fuzzy open cover of X has a finite fuzzy subcover.

CONCLUSION

The investigation into fuzzy topological spaces was given in this article. After gaining an understanding of the fundamentals of fuzzy topology, one may go on to more in-depth studies of the subject, such as theorems and the proofs of those theorems. Concepts of fuzzy topological spaces can then be applied to the solution of real-world engineering issues.

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