FUZZY, INTUITIONISTIC, AND NEUTROSOPHIC SOFT TOPOLOGICAL SPACES: A REVIEW

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Abstract

The idea of fuzzy sets was developed to deal with object uncertainty. The extension of fuzzy sets includes fuzzy topological space, in-tutorial fuzzy sets in topological structure space, vagueness in topological structure space, rough sets in topological space, theory of hesitation, neutrosophic topological space, etc. A family of parameters known as a "soft set" is also a set. Soft sets are combined with different topological structures to create fuzzy soft topological space, intuitionistic fuzzy soft, and neutrosophic soft topological space. This inspires me to research and write a review on numerous soft set ideas. This work provides a thorough analysis of soft topological spaces in a number of sets, including neutrosophy, fuzzy, and intuitionistic fuzzy sets. Finally, for ease of comprehension and to demonstrate the benefits and drawbacks of various instruments now available in the literature, we compared a few of them.
Keywords: Soft sets, fuzzy soft topological space, intuitionistic fuzzy soft topological space, neutrosophic soft topological space.

Introduction

Molodtsov introduced the idea of soft sets in 1999. This idea was devised to get around the challenge of determining membership for each instance. The world of speech is parameterized by the family of SS. Parameters might take the form of words, numbers, or phrases. The parameterization might be defined by anybody at their convenience. Modeling the uncertainty is made much easier using this method. Additionally, Molodtsov highlighted certain applications of SS, such as stability and operations research, and specified a few fundamental operations.

The authors Shabir and Naz presented the first concept of soft spaces, which is based on the discourse universe and has a predetermined set of criteria. The fact that a soft topological space offers a parameterized family of topological spaces was also shown. The notion of soft set theory has been created by scholars.

Zadeh81 first proposed a fuzzy set in 1965, where each element 'a' in A has a membership value, and A is transferred from the realm of discourse to the range [0, 1]. The idea of fuzzy topology, which meets the three axioms of topology, was later proposed in the literature by Chang24. Chang likewise used the same notation for fuzzy topology that Zadeh used for FS. After a few years, Lowen provided a concept of fuzzy topology that was distinct from Chang's. The fuzzy soft set notion and several fundamental operations were established by Maji et al. Incorporating the fuzzy topology and soft set, Tanay et al. later proposed fuzzy soft topological space and defined the fundamental concepts of FSTS. FSTS was used in a variety of contexts, such as game theory, analysis, etc. Roy has conducted more research on fuzzy soft sets in topological space. The authors have

effectively used FSTS in the actual world.

The rejection of one of the set's objects went unaddressed by FS. Therefore, Atanassov introduced the intuitionistic fuzzy set theory, which is a theory in which each item in a set has both acceptance and rejection with the restriction that the total of acceptance and rejection should not be more than 1 and non-negative. The intuitionistic fuzzy set A is an extension of the fuzzy set, where each element of A is a subset of the universal set and each element has a degree of membership and a degree of non-membership, and each function maps from the domain of discourse to the interval [0, 1]. Researchers expanded the idea by making it more generic and obtained fresh findings by extending it. Later, the concept of intuitionistic fuzzy soft set was established by Maji et al. IFSTS was first introduced by D. Coker, who then expanded on the idea for decision-making.

Atanassov neglected to address the issue of object indeterminacy. Smarandache invented the neutrosophic set notion to solve the problem. The truth, indeterminacy, and falsity values for each element in the neutrosophic set are mappings from the universe of discourse to [0, 1] with the restriction that the sum of the values for truth, indeterminacy, and falsity cannot be more than 3 and must not be less than 0. Many hard issues in statistics, graph theory, physics, image processing, networking, and decision-making that can't be handled by current conventional approaches include those where relationships between the objects involve acceptance, rejection, and indeterminacy.

This theory is also generalized in the literature as neutrosophic soft topology, neutrosophic nano topology, neutrosophic nano ideals topology, neutrosophic support soft set, and neutrosophic soft supra topological spaces in multiple sets, among other terms. Neutrophic soft set notions were introduced by Maji et al. The idea of neutrosophic soft sets was effectively implemented by Maji in areas like as pattern recognition and reasoning. The notion of neutrosophic soft topological space was later introduced by Bera1. The writers listed below are NSTS-developed. This study examines soft sets in several topological spaces in great depth. The benefits and drawbacks of various soft topological spaces are discussed. Finally, a comparison table for traditional soft topological space, FSTS, IFSTS, and NSTS is shown.

Soft Topological Spaces in Various Sets
Fuzzy soft set
Yao78 introduced the idea of a soft fuzzy set in 2008, and this idea was examined to determine the importance of the current soft fuzzy set. Finally, several examples are used to compare FSS relations with soft fuzzy set relations. Cagman23 revised the definition of FSS and investigated some of its characteristics. Last but not least, a fuzzy soft aggre- gation operator is designed for efficient decision process development.

Majumdar46 presented the generalized FSS in 2010. Manjumdar presents a few characteristics of generalized FSS and its applications. Tanay was the first to bring FSTS to the literature. Along with a few other concepts and attributes, the authors also defined the terms "neighbourhood," "family of neighborhood," "interior and closure of FSS," "basis for FSTS," and

"subspace of FSTS." Gunduz. Interior defined by C31; FSTS closing. Additionally, Gunduz presented homeomorphism of FSTS as well as open and closed sets in relation to FSS and continuous mapping in FSS. Discussion also included characteristics of fuzzy soft topological structure. In order to illustrate a practical issue with the grey relation analysis theory, Zhi Kong et al. (2011) explored FSS. The outcome is supported by a few examples. In an FSTS, Mahanta presented and investigated fuzzy soft point and its neighborhood. Separation axioms and the connectedness of the FSTS are addressed in relation to the closure and interior of the FSS. The pre-connected, pre-separated, pre-soft subspace of FSTS is a notion that was created and researched by Abd El-Latif.

Multi-fuzzy soft set is the product of Yang et al. combining the ideas of multi-fuzzy set and soft set. Some theoretical operations, such as union, intersection, and complementary, are also specified. An algorithm was created by Yang et al. employing a multi-fuzzy soft set. Eventually, the decision-making issue is examined utilizing the suggested method. Using fuzzy soft sets, Roy and Maji examined the decision-making issues. By taking into account the maximum value among the scores using the scoring function, they create an algorithm for choosing an item from the discourse universe. Cetkin introduced the notions of anti-chain and isomorphism to FSTS as well as the continuous mappings concept.

Semi linked sets, semi s-connected sets, and semi separated sets were all presented by Kandil in FSTS in 2015. Sabir Hussain researched the soft neighborhood at fuzzy soft points and established the soft pre-open set and soft alpha-open set in FSS.

Added a soft regular open set and continued studying. The links between the previously suggested topics are then given. Abd El-Latif presented the pre-open, pre-closed set of FSTS and examined certain aspects of its pre-regular, pre-normal space. A.M. Abd El-Latif developed the fuzzy connected set, fuzzy separated set, and fuzzy s-connected set in the FSTS.

In 2015, A. Kandil et al. proposed fuzzy soft point based on equivalence classes and said that the union of disjoint connected components may be expressed as the universal fuzzy soft set. Through the use of a fuzzy soft set, G. Kalpana et al. developed fuzzy soft r-open and r-closed mappings, fuzzy soft r-closure, fuzzy soft r-interior, and fuzzy soft r-continuous mapping. Abd El-Latif founded the concepts of "closed soft sets" and "regular, normal space" in FSTS as well as the "open soft sets" and "separation axioms" concepts.

Intuitionistic Fuzzy Soft Set

Here, we present the initialization, extensions and generalization of intuitionistic fuzzy soft set in topological structure. Yang originated the concept of interval - valued IFSS, defined the set theoretic operations and finally decision making problem solved by adopting existing algorithm. Mukherjee proposed and studied a new type of sequence of intuitionistic fuzzy soft multi sets and some of its properties are investigated. Also the increasing, decreasing and convergent sequences of intuitionistic fuzzy soft multi- topological spaces are introduced by Mukherjee. Finally, cluster intuitionistic fuzzy soft multi topological space and their properties are studied. In 2010, Xu presented the concept of IFSS by merging K.Atanassov intuitionistic fuzzy set and soft set. Developed some basic

operations and applying this tool to target the type recognition problem. Jiang et al. combined the two classical methods viz. soft set and interval-valued intuitionistic fuzzy set and produced a new result called interval-valued IFSS. Union, intersection and complement of interval-valued IFSS defined and established some basic properties.

In 2012 Yin et al. introduced further the concept of IFSS. In particular, theoretical operations such as union, intersection and complement, etc. are introduced. Mapping on IFSS introduced and their basic properties also presented. Li et al. proposed the novel notion called IFSTS in the year 2013. The author also defined the interior, closure, base, relative complement and absolute IFSS and IFSTS. Some properties of IFSTS also presented.

In 2013, Agarwal et al. developed the concept of generalized IFSS and this developed a generalized pa- rameter to pool the intuitionistic fuzzy numbers. The author has developed three different algorithms mainly for decision making. One is for in medical diagnosis to compare the intuitionistic fuzzy numbers and remain- ing for measure the similarity, if any in selecting the supplier. Kumud Borgohain38 studied IFSTS and defined intuitionistic fuzzy soft separation axioms, normal space and finally completely normal space of IFSTS. Osmanoglu et al. introduced intuitionistic fuzzy soft finer and coarser topological space, Intuitionistic fuzzy soft discrete topology and intuitionistic fuzzy soft indiscrete topology. Further, soft points and complement of intuitionistic fuzzy soft points and separation axioms of the same introduced and their properties also studied. Cetkin introduced the definition of closure intuitionistic supra fuzzy soft topological space.

In 2014, Bayramov introduced the basic definitions of IFSTS namely, null and absolute IFSTS, interior and closure, associated closure of IFSTS. Some basic properties also investigated. Mukherjee established the notion of intuitionistic fuzzy soft multi topological space for the parameterized family and also established the basic structure of intuitionistic fuzzy soft multi topological structure.

Shuker Mahmood studied and established soft b-closed, soft b-continuous mapping, soft b- closed dis- connectedness of IFSTS. In 2017, Yogalakshmi80 initiated the concept of various compactness of IFSTS, namely almost compact, nearly compact, etc and also studied intuitionistic soft fuzzy filter and intuitionistic soft fuzzy prime filter of IFSTS.

Neutrosophic soft set

The overviews of several NSS studies may be found in this section. In 2012, Maji combined soft set and neutrosophic set to establish the idea of NSS. various fundamental NSS operations, including union, intersection, and complement, are described as well as various NSS features. Said Broumi introduced the idea of generalized NSS in 2013 along with its fundamental concepts and features. On NSS, Deli29 developed the concept of relation. Two distinct NSS are created using the NSS composition. Deli looked at the ideas of equivalency relation, equivalence class, and NSS quotient. Deli used the NSS connection to investigate the decision-making issue. Using the already-existing tool known as NSS, Arockiarani constructed a distance measure and scoring function to offer a decision-making dilemma.

Neutosophic vague soft set, an expanded idea of classical soft set, was first

mentioned in the Al-Quran in 2017. A decision-making issue is provided utilizing the suggested notion to demonstrate its efficacy after defining and studying a few fundamental operations and attributes. In order to assess the medical diagnostic issue utilizing interval-valued FSTS, Parimala et al. devised a method. Some fundamental theoretic operations are also looked at in their study.

Bera first proposed the idea of NSTS. The authors also define interior, closure, base for NSTS, subspace of NSTS, and regular NSTS in the introductory article. Finally, certain NSTS and separation axiom qualities with various features are researched and analyzed.

Neutrophic soft linked and compact topological space, as well as a few features, were presented by Bera in 2018. The idea of continuous mapping on NSTS was finally presented and investigated. The neutrosophic soft point was developed by Gunduz Aras et al. along with the concept of NSS. Finally, a thorough study of NSTS's subspace and separation axioms is conducted. A novel idea was put out by Parimala et al. that combines NSS with hesitation degree and is used just to determine the residual of NSS.

Conclusions

Numerous mathematical and non-mathematical disciplines, including operations research, physics, data science, etc., have applications for topological space. It may be challenging to apply the notion of topology in real-world situations at times because to elemental uncertainties, inconsistencies, and incompleteness of knowledge. Intuitionistic fuzzy soft topological space was developed to address several issues that arise in fuzzy soft topology, and fuzzy soft topological space was first proposed

to overcome the challenge in classical sets that deals with object uncertainty. In certain situations when an object's value is ambiguous, the preceding techniques cannot be used. Neutrosophic set has been developed as a result to cope with the ambiguity, inconsistency, and incompleteness. This essay examines each of these techniques in detail. The benefits and drawbacks of every instrument now in use are explored.

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