A BRIEF INTRODUCTION TO THE MATHEMATICS OF THE ANCIENT INDIANS

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Abstract

The purpose of this article is to provide a summary of the mathematics practiced in ancient India, in addition to a discussion of the relevant sources and potential future research avenues. For this reason, it might be helpful to classify the content into the categories on a more general level: Vedic mathematics, Mathematics from the Jaina tradition, Development of the number system and numerals, The mathematical astronomy tradition.

Vedic mathematics

The Sulvasutras are largely responsible for the corpus of mathematical knowledge that has survived from the Vedic era. The Sulvasutras are compositions that serve as instructions for building the vedis (altars) and agnis (fireplaces), which are necessary for the execution of yajnas, which were a central aspect of Vedic civilization. The Sulvasutras describes the setting up of such platforms with tiles of moderate sizes, of simple shapes like squares, triangles, and occasionally special ones like pentagons. The fireplaces were constructed in a variety of shapes such as falcons, tortoises, chariot wheels, circular troughs with handles, pyres, etc. depending on the context and purpose of the particular yajna. Several of the vedis involved, particularly for the yajnas for special occasions, had proportions of the order of 50 to 100 feet, therefore drawing perpendiculars in that environment was necessary while creating the overall layout. They were aware of the "Pythagoras theorem," and all four of the major Sulvasutras explicitly state the theorem. This was accomplished by using both the method currently taught in schools, which involves the perpendicularity of the line joining the centers of two intersecting circles with the line joining the two points of intersection. The Sulvasutras also describe a number of geometrical constructs and concepts, such as how to turn a square into a circle with an equal area and vice versa, as well as a close approximation to the square root of two (see [4] for some details). Like other Vedic wisdom, the Sulvasutras were only ever passed down orally over a considerable amount of time. Sanskrit commentary on several of the Sulvasutras have also existed, however it is unclear when they were written. It's uncertain when the Sulvasutras' first written versions appeared. Beginning in the second part of the nineteenth century, European academics published text copies with contemporary commentary. Regarding the origins of his study of the Sulvasutras, Thibaut notes that Mr. A.C. Burnell was the first to draw attention to the significance of the Sulvasutras. He said on page 29 of his Catalogue of a Collection of Sanscrit Manuscripts that "we must look to the Sulva'portions of the Kalpasutras for the earliest beginnings among the Brahman. While being very thorough, the present translations still have significant gaps, particularly in the case of Manava Sulvasutra, which is more terse than the others. R.G. Gupta [13], Takao Hayashi, and the current author, as well as maybe others who were unknown to the initial translators, have all revealed new findings. One of the contributing reasons in this regard may be the translators' inadequate mathematical background. To set the mathematical information in the Sulvasutras on a thorough foundation, there is a necessity for a thorough review. Also, there is room for improvement in how the outcomes mentioned in the many Sulvasutras relate to one another.

The Sulvasutras' ceremonial framework lends itself to the question of bridging the ritual and mathematical parts, as well as connecting with other comparable circumstances from other civilizations; for a viewpoint on this, the reader may turn to Seidenberg [36]. Is it possible to find any of the fireplaces from earlier times? is another logical question that arises in the context of the Sulvasutras. It seems from the description of the brick structure that they would not have been able to endure the weather for very long; yet, it should be remembered that the objective did not call for a durable construction. However, one substantial brick platform in the traditional shape of a bird with spread wings has been found during excavations at Singhol in Panjab. It is dated to the second century BCE ([18], p. 79–80 and [25], footnote on page 18), but it significantly deviates from the numerical specifications mentioned in the Sulvasutras.

Finding other sites is still an option, but probably not a particularly fruitful one. In addition to the Sulvasutras, mathematical studies of the Vedas have also been made, with a focus on numerical comprehension. With numerals up to 10,000 appearing, the Rigveda exhibits a notable fascination with numbers for a work with such a large scope, embracing both the spiritual and the secular; see [2] for more information. (However, it should be noted that the numbers were not recorded; the reference here is mostly to number names.) The Yajurveda gives names for powers of 10 up to 1012, and numerous basic mathematical characteristics are discovered to be relevant in different circumstances, such as [25]. There is still room for improvement in our comprehension of the evolution as a whole, which would need both a grasp of mathematics and Vedic sanskrit.

Mathematics from the Jaina tradition

The Jainas have a long history of using mathematics in their daily lives. They were driven by thought rather than rituals, which they detested, and they had a highly developed idea of the world. According to Jaina cosmology, the universe is thought to be a flat plane with concentric annular regions surrounding an innermost circular region with a diameter of 100,000 yojanas known as the Jambudvipa (island of Jambu). The annular regions alternately consist of water and land, with their widths increasing by twofold with each succeeding ring. It should be noted that the Puranas also contain this cosmology. Although though the researchers involved were mostly philosophers rather than mathematicians, the geometry of the circle had a significant part in the whole conversation. In Suryaprajnapti, which is thought to date from the fourth or fifth century BCE (the earliest extant manuscript is from around 1500, on paper), as well as in the writings of Umasvati, who is thought to have lived in the second century CE according to the Digambara tradition of the Jainas and around 150 BCE according to the Svetambara tradition, many properties of the circle are described.

Suryaprajnapti mentions the then-traditional number of 3 for it and discards it in favour of 10. This break from the ancient notion of 3 as the ratio of the circumference to the diameter is one of the distinctive aspects of the Jaina tradition. The Jainas were also aware of the fact that the ratio of the circumference to the diameter and the area of a circle to the square of its radius are the same. Also, they had intriguing approximations for the lengths of circular arcs, the areas they occupied, and the related chord.

It is unclear how they arrived at these formulas. Other mathematical themes with in-depth study may be found in Jaina literature include permutations and combinations, sequences, and categorization of infinities. From the eighth century forward, there is clear evidence of mathematical work in the Jaina tradition, and it may have persisted up to the middle of the fourteenth century. One of the well-known works in this regard is Mahavira's Ganita-sarasangraha, which was composed in 850. Regarding the development of mathematics in the Jaina canon, additional individuals that may be cited include Virasena (8th century), Sridhara (between 850 and 950), Nemicandra (about 980 CE), and T. hakkura Pheru (14th century).

One may detect a fusion of the indigenous Jaina heritage with Indo-Persian literature in the early 14th century works of Thakkura Pheru. A portion of the geometry mentioned, which involves domes, arches, and other structures, is closely related to the growth of Islamic architecture in India. Mahendra Suri, a Jaina astronomer who worked at the court of a Tughluq dynasty ruler in the late 14th century, wrote about the astrolabe. Jaina astronomical writings have a number of linkages between Sanskrit and Islamic science, as detailed in [26]. Rangacharya published the Ganita-sarasam graha text with an English translation in 1912, and it was recently reissued [31].

In more recent times, Padmavathamma has published an edition that includes the original text along with translations into English and Kannada [22]. Ganita-sara-kaumudi by Thakkura Pheru has lately been published thanks to a cooperative effort [34]. [14] has a series of articles outlining many facets of Jaina mathematics; also, [5] provides an earlier discussion of the subject. Overall, however, the systematic examination of the Jaina texts from a mathematical perspective has not been done in sufficient depth. There is a significant shortage of knowledge on the ancient Jaina writings from the BCE and early centuries of the CE. The first step is to gather enough information about the resources that are accessible.

Development of the number system and numerals

Buddhist, Jaina, and Vedic traditions have all been studied in relation to the evolution of the number system in India. As we said in the prior parts on the Vedic and Jaina traditions, India has long had a fascination with enormous numbers. Tallakshana, a phrase from the Buddhist past, stood for 1053, and Buddha himself was famed for his skill with numbers. Big numbers are also mentioned in the Buddhist tradition. Nevertheless, the names of the powers of ten varied depending on the tradition and the time period; for instance, Parardha, which literally translates as "half-way to heaven," stood for 1012 in early literature but 1017 in later works like those of

Bhaskara II. The oral practise of using various powers of ten is believed to have been a major factor in the subsequent development of decimal representation in writing, which seems to have occurred in the first centuries of the common period. This connection is not very clear, though, because there was a significant interval of time—a few hundred years—during which written forms of numbers did not correspond to place value notation (see below). In addition, even after the decimal place value system with zero gained popularity, the other systems appear to have been in use for a considerable amount of time. The oral tradition often uses allusions to powers of 10, and the seeming simplicity and beauty of the decimal place value system raise questions regarding the reasons behind this. Nonetheless, the Chinese seem to have started using the decimal place value system for representing numbers at least as early as the third century BCE. They did not use a sign for zero, instead leaving a blank space in its place. As far as whole numbers are concerned, the introduction of zero as a placeholder paved the way for how we now write numbers. The full decimal representation system that we now use, which includes the fractional part and has a separating decimal point, had its beginnings in 15th-century Europe, though it is noted to have been first used by Arabs in the 10th century. Early common period in India saw the conceptualization of zero as a number, incorporated into the number system, and in Brahmagupta's book Brahmasphut asiddhanta we have a systematic presentation, in the seventh century, which also covers arithmetic with negative numbers. The evolution of the numbers is a related subject.

Many written numbers have been researched. The oldest of these would have been in the form of Indus seals, where the strokes represented numerals. Some of the ancient numerals include the Kharosthi numerals, which were in use between the third century BCE and the third century CE and can be found in the inscriptions from Kalderra, Taksasila, and Lorian, as well as the Brahmi numerals from Naneghat (first century BCE), which, incidentally, did not employ the place value system. The first known inscriptions using the decimal numera system are believed to be from Gujarat and date to 595 CE, however R. Saloman claims in [32] that this is a fake inscription. The earliest zero in an Indian inscription dates to 876 CE and was discovered in a temple near Gwalior (an image of this may be viewed online, thanks to Bill Casselman).

Bhagwanlal Indra ji conducted a great quantity of study in the late nineteenth century, which is detailed in his book by

George Ifrah [16]; (a good deal of what Ifrah says has been contradicted by various reviewers - see [8] for details - one may nevertheless suppose that what he reports from the work of Bhagwanlal Indra ji would be reliable). In addition to the inscriptions in stone, the presence of decimal numbers has been investigated on copper plates that served as legal records from the 7th to the 10th century, detailing grants of gifts by monarchs or wealthy individuals to Brahmanas. As a result of attempts to misappropriate the properties involved, there have been some objections to this source due to the plates' forgery susceptibility; however, while this may apply to a few plates, as a whole, the plates may not be discounted as a source; for a discussion on this, see [7], pages 44–48, where the author attempts to refute the objections. Another source in this regard are the numbers found in old texts. Large numbers are used in mathematical works in many traditions, thus it would be fascinating to analyse how these numbers are represented in the existent manuscripts from various eras. The origin of these systems would also be a related problem given that different Indian languages have their unique symbols for the particular numbers. The author is not aware of any in-depth research on the subject. This content should be systematically archived from a variety of sources, and then the sources should be examined to determine the direction in which the ideas developed.

The mathematical astronomy tradition

From the third or fourth century, when it began to flourish, the Siddhanta or mathematical astronomy tradition, has dominated the field of mathematics in India. This legacy has been practically constant. The first significant figure in the tradition, Aryabhata (476–550), is credited with founding scientific astronomy in India. Despite the fact that Bhaskara II (1114–1185) is regarded as the final major exponent in the continuity, the Siddhanta tradition did continue after him.

The Aryabhatiya, which was composed in 499 and is the oldest full Siddhanta text still existing, is fundamental to the tradition as well as to the subsequent works of the Kerala school of Madhava, which I will explore below. There are four chapters, numbered Gitikapada, Ganitapada, Kalakriyapada, and Golapada, totaling 121 verses. In a single stanza, the first one lays out the cosmology and includes a table of 24 sine differences spaced at 225 minute arc intervals. The second chapter, as its name implies, is devoted to mathematics and contains, among other things, methods for finding square and cube roots, a rough expression for, formulae for the areas and volumes of various geometric figures, formulae for the sums of consecutive integers, formulae for the sums of squares, formulae for the sums of cubes, and formulae for computation of interest. For more information, see [40] and [41]. The next two chapters deal with astronomy and topics like eclipses, planet distances and relative movements, etc (we shall not go into the details here).

Throughout the time up till Bhaskara,Narayana, some of the significant individuals were Varahamihira, Bhaskara, Brahmagupta, Govindaswami, Sankaranarayana Aryabhat, Vijayanandi, Sripati, and Brahmadeva. e'sa may have received the names Pandit and Gan in subsequent years straight from the oral tradition. It should be noted that many of the dates given here are approximations since there isn't any trustworthy historical data on them and the dating is dependent on a number of indirect assumptions.

Astronomy makes up the majority of the work in Brahmagupta's Brahmasphutasiddhanta. General mathematics is covered in two chapters, the 12th and the 18th. In addition, rather of the mathematical subjects covered in the works, the 21st chapter has poems on trigonometry, which in Siddhanta astronomy literature used to be mixed with astronomy. Chapter 11, which is a criticism of prior works like Aryabhatiya, is another unique aspect of the text. Like other scientific groups, this tradition had many internal disagreements; the harsh language employed in this chapter, however, would be unsettling to modern readers.

A sytematic description of arithmetic operations, including those involving negative integers, is found in Chapter 12, which was unknown to European mathematics until the middle of the second century. The chapter also covers geometry, with a focus on his well-known formula for the area of a quadrilateral, which generalises Heron's formula for the area of a triangle. However, this formula is presented without the requirement of the quadrilateral's cyclicity, which later mathematicians in the tradition criticised. The 18th chapter focuses on Astronomy makes up the majority of the work in Brahmagupta's Brahmasphutasiddhanta. General mathematics is covered in two chapters, the 12th and the 18th. In addition, rather of the mathematical subjects covered in the works, the 21st chapter has poems on trigonometry, which in Siddhanta astronomy literature used to be mixed with astronomy.

Chapter 11, which is a criticism of prior works like Aryabhatiya, is another unique aspect of the text. Like other scientific societies, this tradition also saw numerous internal conflicts, but the harsh language employed in this chapter would be unsettling by modern standards. A sytematic description of arithmetic operations, including those involving negative integers, is found in Chapter 12, which was unknown to European mathematics until the middle of the second century. The chapter also covers geometry, with a focus on his well-known formula for the area of a quadrilateral, which generalises Heron's formula for the area of a triangle. However, this formula is presented without the requirement of the quadrilateral's cyclicity, which later mathematicians in the tradition criticised. The kuttaka and other techniques for resolving second-degree indeterminate equations are covered in the 18th chapter. Readers are directed to [39] for the original text and, for example, [25] for a synopsis of the contents. In the Arab East, mathematics was greatly influenced by the Brahmsphutasiddhanta.

Another crucial responsibility in this regard is the cataloguing and accessibility of the existing manuscripts. The role and proportion of mathematics in the body as a whole seem to be difficult to determine and call for serious study. A good beginning may be said to have been made in [20], where it is noted in particular that only a small proportion of the listed manuscripts from the census pertain directly to mathematics - the census encompasses all manuscripts with some coherence in the exact sciences. K.V. Sarma published a similar compilation on the Sanskrit literature in the repositories in Tamilnadu and Kerala in 2002. S.N. Sen, A.K. Bag, and R.S. Sarma published a bibliography of Sanskrit books on mathematics and astronomy in 1966 [38]. Understanding the equipment utilised, such as clocks, astrolabes, and other devices used in astrology, as well as the mathematics behind them, is a relevant issue that has to be addressed. S.R. Sarma has put forth a lot of effort in this regard. Although it is often held that mathematical astronomy eclipsed the study of astronomy in India, research is also needed on the interaction between observational and mathematical components.

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