APPLICATION OF THE NONLINEAR DIFFERENTIAL METHOD TO NONLINEAR ORDINARY DIFFERENTIAL EQUATIONS

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Abstract

In this study, we investigate a brand-new technique called the Natural Decomposition Method. For three distinct kinds of nonlinear ordinary differential equations, we employ the natural decomposition method to provide precise solutions (NLODEs). The Natural Transform Method (NTM) and the Adomian Decomposition Method serve as the foundation for the natural decomposition METHOD. We effectively treat a class of nonlinear ordinary differential equations using the novel methodology in a straightforward and beautiful manner. The suggested approach provides precise answers as a quick convergence series. To solve linear and nonlinear differential equations, the Natural Decomposition Method (NDM) is a superb mathematical tool. The natural decomposition method is effective and simple toutilise, one may infer.

Keywords *Natural transform, Laplace transform, ordinary differential equations.*

1. Introduction

Due to their many applications, nonlinear differential equations have attracted a lot of attention. In many areas of practical and pure mathematics, including engineering, applied mechanics, quantum physics, analytical chemistry, astronomy, and biology, nonlinear ordinary differential equations are crucial. Researchers have been focusing on analytical and numerical solutions to nonlinear ordinary differential equations over the last 10 years. It is crucial to be knowledgeable about both established and cutting- edge techniques for solving both linear and nonlinear ordinary differential equations. The Natural Decomposition Method (NDM), a novel integral transform technique, is introduced here and used to precisely solve nonlinear ODEs [29]. There are several integral transform techniques [3, 13–19] that may be used to solve ODEs. The Laplace transformation is the one that is used the most [30]. Other techniques, such the Sumudu transform [6], Reduced Differential Transform Method (RDTM) [25-28], and Elzaki transform [14-19], have recently been employed to solve PDEs and ODEs. The N-Transform was utilised by Fethi Belgacem and R. Silambarasan [11, 12] to solve the Maxwell's equation, the Bessel differential equation, the linear and nonlinearKlein Gordon Equations, among other problems. Additionally, Zafar H. Khan and Waqar A. Khan [21] solved linear differential equations using the N-Transform, and they provided a table with some characteristics of the N-Transform for various functions.

To demonstrate the effectiveness and precision of the NDM, we give a number of examples from the physics and engineering domains. George Adomian's Adomian decomposition technique (ADM) [1,2] has been used to solve a large class of linear and nonlinear PDEs. The NATURAL DECOMPOSITION METHOD provides precise answers and analytically approximate solutions that quickly converge to the exact solutions for nonlinear models with dependable results.

The purpose of this study is to create an effective algorithm for numerical computing using the natural decomposition approach. The answer is provided as a fast converging series via the natural decomposition approach.

We resolve the following NLODEs in this paper:

The nonlinear second order differential equation has the following form:

(1.1)
$$\frac{d^2v}{dt^2} + \left(\frac{dv}{dt}\right)^2 + v^2(t) = 1 - \sin(t),$$

subject to the initial conditions

$$v(0) = 0, \qquad v'(0) = 1. \quad (1.2)$$

Second, the first order nonlinear ordinary differential equation of the form:

$$\frac{dv}{dt} - 1 = v^2(t), \quad (1.3)$$

subject to the condition

v(0) = 0. **(1.4)**

Third, the nonlinear Riccati differential equation of the form:

$$\frac{dv}{dt} = 1 - t^2 + v^2(t), \quad (1.5)$$

Subject to the condition

$v(0) = 0.$ (1.6)

The remainder of this essay is structured as follows: We provide some background information regarding the NATURAL DECOMPOSITION METHODin Sections 2 and 3. We describe the NATURAL DECOMPOSITION METHODapproach in section 4. To demonstrate the viability of our approach, we apply the NATURAL DECOMPOSITION METHODto three test issues in section 5. The discussion and conclusion of this essay are found in Section 6.

2. Basic Idea of The Natural Transform Method

In this section, we present some background about the nature of the Natural Transform Method (NTM). Assume we have a function $f(t)$, $t \in (-\infty,\infty)$, and then the general integral transform is defined as follows [11, 12]:

$$\Im\left[f(t)\right](s) = \int_{-\infty}^{\infty} K(s,t)\, f(t)\, dt, \quad (2.1)$$

K(s, t) denotes the transform's kernel, and s is a real (complex) integer that is independent of t. Be aware that Eq. (2.1) yields the Laplace transform, Hankel transform, and Mellin transform, respectively, when K(s, t) is est, t Jn(st), and ts1(st).

Now, for $f(t)$, $t \in (-\infty,\infty)$ consider the integral transforms defined by:

$$\Im\left[f(t)\right](u) = \int_{-\infty}^{\infty} K(t)\, f(ut)\, dt, \tag{2.2}$$

And

$$\Im\left[f(t)\right](s,u) = \int_{-\infty}^{\infty} K(s,t)\, f(ut)\, dt. \tag{2.3}$$

It is important to note that, when K(t) = et, Eq. (2.2) offers the integral Sumudu transform with u in lieu of the original parameter s. Additionally, the generalised Laplace and Sumudu transforms are defined by [11, 12] for any value of n, respectively

$$\ell\left[f(t)\right] = F(s) = s^{n} \int_{0}^{\infty} e^{-s^{n+1}t} f(s^{n}t)\, dt, \tag{2.4}$$

And

$$\mathbb{S}\left[f(t)\right] = G(u) = u^{n} \int_{0}^{\infty} e^{-u^{n}t} f(tu^{n+1})\, dt. \tag{2.5}$$

Note that when $n = 0$, Eq. (2.4) and Eq. (2.5) are the Laplace and Sumudu transform, respectively.

3. De_nitions and Properties of the N{Transform

The natural transform of the function $f(t)$ for $t \in (-\infty,\infty)$ is defined by [11, 12]:

$$\mathbb{N}\left[f(t)\right]| = R(s,u) = \int_{-\infty}^{\infty} e^{-st} f(ut)\, dt; \quad s,\, u \in (-\infty,\infty), \tag{3.1}$$

where the variables s and u are the natural transform variables and N[f(t)] is the natural transformation of the time function f(t). Note that Eq. (3.1) has the following possible writing formats: [4, 5]:

$$\begin{aligned}
\mathbb{N}\left[f(t)\right] &= \int_{-\infty}^{\infty} e^{-st} f(ut)\, dt; \quad s,\, u \in (-\infty,\infty) \\
&= \left[\int_{-\infty}^{0} e^{-st} f(ut)\, dt;\ s,\, u \in (-\infty,0)\right] + \left[\int_{0}^{\infty} e^{-st} f(ut)\, dt;\ s,\, u \in (0,\infty)\right] \\
&= \mathbb{N}^{-}\left[f(t)\right] + \mathbb{N}^{+}\left[f(t)\right] \\
&= \mathbb{N}\left[f(t)H(-t)\right] + \mathbb{N}\left[f(t)H(t)\right] \\
&= R^{-}(s,u) + R^{+}(s,u),
\end{aligned}$$

where the Heaviside function is H(.). The Natural transform (N-Transform) is defined on the set if and only if the function f(t)H(t) is defined on the positive real axis with t∈ R

$$A = \left\{ \begin{array}{l} f(t) : \exists\ M,\ \tau_1,\ \tau_2 > 0,\ \text{such that}\ |f(t)| < M e^{\frac{|t|}{\tau_j}}, \\ \quad \text{if}\ t \in (-1)^j \times [0,\infty),\ j \in \mathbb{Z}^+ \end{array} \right\}$$

$$\mathbb{N}[f(t)H(t)] = \mathbb{N}^+[f(t)] = R^+(s,u) = \int_0^\infty e^{-st} f(ut)\,dt; \quad s,\ u \in (0,\infty), \tag{3.2}$$

where the Heaviside function is H(.). Be aware that Equation (3.2) may be simplified to the Laplace transform if u = 1 and to the Sumudu transform if s = 1. We now provide a few N Transforms together with their Sumudu and Laplace conversions [11,12].

$f(t)$	$\mathbb{N}[f(t)]$	$\mathbb{S}[f(t)]$	$\ell[f(t)]$
1	$\frac{1}{s}$	1	$\frac{1}{s}$
t	$\frac{u}{s^2}$	u	$\frac{1}{s^2}$
e^{at}	$\frac{1}{s-au}$	$\frac{1}{1-au}$	$\frac{1}{s-a}$
$\frac{t^{n-1}}{(n-1)!}, n=1,2,\ldots$	$\frac{u^{n-1}}{s^n}$	u^{n-1}	$\frac{1}{s^n}$
$\sin(t)$	$\frac{u}{s^2+u^2}$	$\frac{u}{1+u^2}$	$\frac{1}{1+s^2}$

Table 1. Special N{Transforms and the conversion to Sumudu and Laplace

Remark 3.1. More information on the Natural transform may be found in [11, 12]. As of right now, the following N-Transforms' crucial characteristics are mentioned [11, 12, 20, 21]:

Table 2. Properties of N Transforms

Functional Form	Natural Transform
$y(t)$	$Y(s,u)$
$y(at)$	$\frac{1}{a}Y(s,u)$
$y'(t)$	$\frac{s}{u}Y(s,u) - \frac{y(0)}{u}$
$y''(t)$	$\frac{s^2}{u^2}Y(s,u) - \frac{s}{u^2}y(0) - \frac{y'(0)}{u}$
$\gamma\, y(t) \pm \beta\, v(t)$	$\gamma\, Y(s,u) \pm \beta\, V(s,u)$

4. The Natural Decomposition Method

We demonstrate the Natural Decomposition Method's application to a few nonlinear ordinary differential equations in this section.

Methodology of the NDM:

Consider the general nonlinear ordinary differential equation of the form:

***Lv* + *R*(*v*) + *F*(*v*) = *g*(*t*),** **(4.1)**

subject to the initial condition

***v*(0) = *h*(*t*),** **(4.2)**

where g(t) is the nonhomogeneous term, F(v) is the nonlinear term, R is thedifferential operator's residual, L is an operator of the highest derivative, and
If L is a first-order differential operator, then the following results from applying theN-Transform to Eq. (4.1):

$$\frac{sV(s,u)}{u} - \frac{V(0)}{u} + \mathbb{N}^{+}[R(v)] + \mathbb{N}^{+}[F(v)] = \mathbb{N}^{+}[g(t)]. \quad (4.3)$$

By substituting Eq. (4.2) into Eq. (4.3), we obtain:

$$V(s,u) = \frac{h(t)}{s} + \frac{u}{s}\mathbb{N}^{+}[g(t)] - \frac{u}{s}\mathbb{N}^{+}[R(v) + F(v)]. \quad (4.4)$$

Taking the inverse of the N–Transform of Eq. (4.4), we have:

$$(4.5) \quad v(t) = G(t) - \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}[R(v) + F(v)]\right],$$

where $G(t)$ is the source term. We now assume an infinite series solution of the unknown function $v(t)$ of the form:

$$(4.6) \quad v(t) = \sum_{n=0}^{\infty} v_n(t).$$

Then by using Eq. (4.6), we can re-write Eq. (4.5) in the form:

$$\sum_{n=0}^{\infty} v_n(t) = G(t) - \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[R\sum_{n=0}^{\infty} v_n(t) + \sum_{n=0}^{\infty} A_n(t)\right]\right], \quad (4.7)$$

where $An(t)$ is an Adomian polynomial which represent the nonlinear term. Comparing both sides of Eq. (4.7), we can easily build the recursive relation as follows:

Eventually, we have the general recursive relation as follows:

$$\begin{aligned} v_0(t) &= G(t), \\ v_1(t) &= -\mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}[Rv_0(t) + A_0(t)]\right], \\ v_2(t) &= -\mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}[Rv_1(t) + A_1(t)]\right], \\ v_3(t) &= -\mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}[Rv_2(t) + A_2(t)]\right]. \end{aligned}$$

$$(4.8) \quad v_{n+1}(t) = -\mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}[Rv_n(t) + A_n(t)]\right], \qquad n \geq 0.$$

Hence, the exact or approximate solution is given by:

$$v(t) = \sum_{n=0}^{\infty} v_n(t). \tag{4.9}$$

5. Worked Examples

In this part, we apply the NATURAL DECOMPOSITION METHODto three practical applications and contrast our findings with the precise solutions already in existence.

Example 5.1. Consider the first order nonlinear differential equation of the form:

$$\frac{d^2v}{dt^2} + \left(\frac{dv}{dt}\right)^2 + v^2(t) = 1 - \sin(t), \tag{5.1}$$

subject to the initial condition

$$v(0) = 0, \qquad v'(0) = 1. \tag{5.2}$$

We begin by taking the N–transform to both sides of Eq. (5.1), we obtain:

$$\frac{s^2V(s,u)}{u^2} - \frac{sV(0)}{u^2} - \frac{v'(0)}{u} + \mathbb{N}^+\left[\left(\frac{dv}{dt}\right)^2\right] + \mathbb{N}^+\left[v^2(t)\right] = \frac{1}{s} - \frac{u}{s^2+u^2}. \tag{5.3}$$

By substituting Eq. (5.2) into Eq. (5.3) we obtain

$$V(s,u) = \frac{u^2}{s^3} + \frac{u}{s^2+u^2} - \frac{u^2}{s^2}\mathbb{N}^+\left[\left(\frac{dv}{dt}\right)^2 + v^2(t)\right]. \tag{5.4}$$

Then by taking the inverse N–Transform of Eq. (5.4), we have:

$$v(t) = \frac{t^2}{2!} + \sin(t) - \mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^+\left[\left(\frac{dv}{dt}\right)^2 + v^2(t)\right]\right]. \tag{5.5}$$

We now assume an infinite series solution of the unknown function *v*(*t*) of the form:

$$v(t) = \sum_{n=0}^{\infty} v_n(t). \tag{5.6}$$

By using Eq. (5.6), we can re-write Eq. (5.5) as follows:

$$\sum_{n=0}^{\infty} v_n(t) = \frac{t^2}{2!} + \sin(t) - \mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^+\left[\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} B_n\right]\right], \tag{5.6}$$

where *An* and *Bn* are the Adomian polynomials of the nonlinear terms $\left(\frac{dv}{dt}\right)^2$ and $v^2(t)$ respectively.

Then by comparing both sides of Eq. (5.7), we can drive the general recursive relation as follows:

$$
\begin{aligned}
v_0(t) &= \frac{t^2}{2!} + \sin(t), \\
v_1(t) &= -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^+\left[A_0 + B_0\right]\right], \\
v_2(t) &= -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^+\left[A_1 + B_1\right]\right], \\
v_3(t) &= -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^+\left[A_2 + B_2\right]\right].
\end{aligned}
$$

Therefore, the general recursive relation is given by:

$$v_{n+1}(t) = -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^+\left[A_n + B_n\right]\right], \quad n \geq 0. \tag{5.8}$$

Then by using the recursive relation derived in Eq. (5.8), we can easily compute the remaining components of the unknown function *v*(*t*) as follows:

$$
\begin{aligned}
v_1(t) &= -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^+\left[A_0 + B_0\right]\right] \\
&= -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^+\left[(v_0')^2 + v_0^2\right]\right] \\
&= -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^+\left[(v_0')^2 + v_0^2\right]\right] \\
&= -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^+\left[1\right]\right] + \cdots \\
&= -\mathbb{N}^{-1}\left[\frac{u^2}{s^3}\right] + \cdots \\
&= -\frac{t^2}{2!} + \cdots .
\end{aligned}
$$

In order to demonstrate that the non-canceled term of v0(t) still solves the given differential equation, one may cancel the noise terms that emerge between v0(t) and v1(t), which results in an exact solution of the form:

***v*(*t*) = sin(*t*).**

The exact solution is in closed agreement with the result obtained by (ADM) [31].**Example 5.2.** Consider the first order nonlinear ordinary differential equation of the form [31]:

$$\frac{dv}{dt} - 1 = v^2(t), \quad (5.9)$$

subject to the initial condition

$$v(0) = 0. \quad (5.10)$$

$$\frac{s}{u}V(s,u) - \frac{1}{u}V(s,u) - \frac{1}{s} = \mathbb{N}^+\left[v^2(t)\right].$$

Taking the Natural transform to both sides of Eq. (5.9), we obtain:

Substituting Eq. (5.10), we obtain:

$$V(s,u) = \frac{u}{s^2} + \frac{u}{s}\left[\mathbb{N}^+\left[v^2(t)\right]\right]. \quad (5.11)$$

Taking the inverse Natural transform of Eq. (5.12), we obtain:

(5.12)

$$v(t) = t + \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[v^2(t)\right]\right]\right]. \quad (5.13)$$

We now assume an infinite solution of the unknown function $v(t)$ of the form:

$$v(t) = \sum_{n=0}^{\infty} v_n(t). \quad (5.14)$$

Using Eq. (5.14), we can re-write Eq. (5.13) in the form:

$$\sum_{n=0}^{\infty} v_n(t) = t + \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[\sum_{n=0}^{\infty} A_n(t)\right]\right]\right], \quad (5.15)$$

where $An(t)$ is the Adomian polynomial representing the nonlinear term $v2(t)$.Then from Eq. (5.15), we can generate the recursive relation as follows:

$$v_0(t) = t,$$

$$v_1(t) = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[A_0(t)\right]\right]\right],$$

$$v_2(t) = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[A_1(t)\right]\right]\right],$$

$$v_3(t) = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[A_2(t)\right]\right]\right].$$

Thus, the general recursive relation is given by:

$$v_{n+1}(t) = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[A_n(t)\right]\right]\right], \qquad n \geq 0. \quad (5.16)$$

Using Eq. (5.16), we can easily compute the remaining components of the unknown function $v(t)$ as follows:

$$v_1(t) = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[A_0(t)\right]\right]\right] = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[v_0^2(t)\right]\right]\right]$$
$$= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[t^2\right]\right]\right] = \mathbb{N}^{-1}\left[\frac{2u^3}{s^4}\right] = \frac{1}{3}t^3,$$
$$v_2(t) = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[A_1(t)\right]\right]\right] = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[2v_0(t)v_1(t)\right]\right]\right]$$
$$= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[\frac{2t^4}{3}\right]\right]\right] = \mathbb{N}^{-1}\left[\frac{48u^5}{3s^6}\right] = \frac{2t^5}{15},$$
$$v_3(t) = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[A_2(t)\right]\right]\right] = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[2v_0(t)v_2(t) + v_1^2(t)\right]\right]\right]$$
$$= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[\frac{17t^6}{45}\right]\right]\right] =|\mathbb{N}^{-1}\left[\frac{12240u^7}{45s^8}\right] = \frac{17t^7}{315}.$$

Then the approximate solution of the unknown function *v*(*t*) is given by:

$$v(t) = \sum_{n=0}^{\infty} v_n(t)$$
$$= v_0(t) + v_1(t) + v_2(t) + v_3(t) + \cdots$$
$$= t + \frac{1}{3}t^3 + \frac{2t^5}{15} + \frac{17t^7}{315} + \cdots$$

Hence, the exact solution of Eq. (5.9) is given by:

v(*t*) = tan(*t*).

The exact solution is in closed agreement with the result obtained by (ADM) [31].

Example 5.3. Consider the Riccati differential equation of the form [31]:

$$\frac{dv}{dt} = 1 - t^2 + v^2(t),$$

(5.17)

subject to the initial condition

v(0) = 0 **(5.18)**

$$\frac{sV(s,u)}{u} - \frac{v(0)}{u} = \frac{1}{s} - \frac{2u^2}{s^3} + \mathbb{N}^+\left[v^2(t)\right].$$

Taking the N–Transform to both sides of Eq. (5.17), we obtain:

(5.19)

By substituting Eq. (5.18) into Eq. (5.19), we obtain:

$$v(s,u) = \frac{u}{s^2} - \frac{2u^3}{s^4} + \frac{u}{s}\mathbb{N}^+\left[v^2(t)\right].$$

(5.20)

Taking the inverse N–Transform of Eq. (5.20), we have:

$$v(t) = t - \frac{t^3}{3} + \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^+\left[v^2(t)\right]\right].$$

(5.21)

We now assume an infinite series solution of the unknown function *v*(*t*) of the form:

$$v(t) = \sum_{n=0}^{\infty} v_n(t).$$

(5.22)

Then by using Eq. (5.22), we can re-write Eq. (5.21) in the form:

$$\sum_{n=0}^{\infty} v_n(t) = t - \frac{t^3}{3} + \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[\sum_{n=0}^{\infty} A_n(t)\right]\right], \quad \textbf{(5.23)}$$

where *An* is the Adomian polynomial which represent the nonlinear term *v*2(*t*). By comparing both sides of Eq. (5.23), we can easily build the general recursiverelation as follows:

$$v_0(t) = t - \frac{t^3}{3},$$
$$v_1(t) = \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[A_0(t)\right]\right],$$
$$v_2(t) = \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[A_1(t)\right]\right],$$
$$v_3(t) = \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[A_2(t)\right]\right].$$

$$v_{n+1}(t) = \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[A_n(t)\right]\right].$$

Then the general recursive relation is given by:

(5.24)

By using Eq. (5.24), we can easily compute the remaining components of theunknown function *v*(*t*) as follows:

$$\begin{aligned} v_1(t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[A_0(t)\right]\right] \\ &= \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[v_0^2(t)\right]\right] \\ &= \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[\left(t-\frac{t^3}{3}\right)^2\right]\right] \\ &= \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[t^2\right]\right] - \frac{2}{3}\mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[t^4\right]\right] + \frac{1}{9}\mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[t^6\right]\right] \\ &= \mathbb{N}^{-1}\left[\frac{2u^3}{s^4}\right] - \frac{2}{3}\mathbb{N}^{-1}\left[\frac{4!u^5}{s^6}\right] + \frac{1}{9}\mathbb{N}^{-1}\left[\frac{6!u^7}{s^8}\right] \\ &= \frac{t^3}{3} - \frac{2t^5}{15} + \frac{t^7}{63}. \end{aligned}$$

It is clear from v1(t) that the components of v0 include one noise term (t). The remaining non-canceled component in v0(t), after eliminating the noise term, gives us the precise answer. This is simple to confirm by replacement.

As a result, the following provides the precise answer to the given problem:

***v*(*t*) = *t*.** **(5.25)**

The exact solution is in closed agreement with the result obtained by (ADM) [31].

6. Conclusion

In this study, two nonlinear ordinary differential equations and the Riccati differential equation were solved using the Natural Decomposition Method (NDM). We were able to solve all three applications precisely. In comparison to current approaches, the NATURAL DECOMPOSITION METHOD brings a major improvement in the areas. Future work will focus on applying the NATURAL DECOMPOSITION METHODto additional linear nonlinear differential equations(PDEs, ODEs) that come up in other branches of engineering and research.

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